A Study of Neutrino Oscillations
Disentangling CP violation, Matter Effect, and Mass Hierarchy w

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Undergraduate Honours Thesis

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Abstract

This thesis investigated the effects of CP-violation, matter effect, and mass hierarchy in neutrino oscillations. In particular, the affect of these effects was shown to, in principle, be seen using an experiment like the Tokai to Kamioka (T2K) experiment. At T2K, they send a beam of ν_μ through matter to a detector which is 295 km away (a baseline of 295 km). It was found that by having a neutrino beam with a range of energies should help disentangle these effects, but that a longer baseline is preferable.

1 Introduction

Neutrinos are electrically neutral, subatomic particles that were first postulated by Pauli in 1930 to conserve energy and momentum in beta decay. Neutrinos only interact via the weak force, making them very difficult to detect. There are three different flavours of neutrinos: electron neutrinos, muon neutrinos and tau neutrinos where the flavour indicates the type of lepton which is involved in weak interactions with the neutrino.

Neutrinos are now a very actively researched branch of physics because they were found to oscillate from own flavour to another and therefore not massless, which was not accounted for by the Standard Model of particle phyics. Neutrino oscillations helped explain the solar neutrino problem, where only about a third of the expected neutrinos that should be coming from the sun were detected. The neutrino oscillation in vacuum happens because the neutrino mass eigenstates and the flavor eigenstates are not the same.

Neutrinos, though weakly interacting, do interact with matter to varying degrees. When they pass through matter, the Mikheyev-Smirnov-Wolfenstein (MNS) effect occurs, modifying the oscillations due to the neutrinos now having a different effective mass (also called the matter effect). Neutrino physics is currently a fast growing branch of physics with experiments worldwide that are looking to precisely measure the neutrino oscillation parameters.

Some of the goals of neutrino physics experiments will be reviewed in this thesis. In particular determining the neutrino mass hierarchy, whether there is any CP-violation in the neutrino sector, and determining whether the neutrino is its own anti-particle (Majorana) will be investigated.

2 Neutrino Oscillations in Vacuum

Neutrinos have been experimentally determined to have small non-zero masses. Neutrinos interact only by the weak force and are governed by the W and Z bosons. Neutrinos are found to be left-handed and antineutrinos right-handed. When a neutrino goes through an electroweak interaction it does so in a particular flavour state. Since the neutrino flavour state is not the same as its mass state, when the neutrino propagates it can change from one flavour to another. Since the neutrino can change flavour as it propagates, neutrino flavour is not conserved when neutrinos travel over macroscopic distances.

A neutrino with a particular flavor can be described by the state

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} U^*_{\alpha k}|\nu_k\rangle, \tag{1}$$

where α can be e, μ, or τ to represent the flavour, and k = 1,2,3 represents the mass. U is the unitary Maki-Nakagawa-Sakata (MNS) leptonic mixing matrix [2]. Neutrinos can be described as wave packets, but neutrino oscillations can be described using a plane wave description since neutrinos are travelling at relativistic speeds. Therefore the neutrino state evolves according to the time-dependent Schrodinger Equation

$$|\nu_\alpha(x,t)\rangle = \sum_{\beta=e,\mu,\tau} e^{iE_k t + ip_k x}|\nu_k|. \tag{2}$$

By taking neutrinos to be ultra-relativistic, one can make the assumptions that $t \simeq x$ and $E_k^2 \simeq p_k^2 + m_k^2$. Then by substitution and by taking L to be the base path length, equation 1 becomes

$$|\nu_\alpha(L)\rangle = \sum_{\beta=e,\mu,\tau} (\sum_k U_{\alpha k} e^{-im_k^2 L/2E} U^*_{\beta k})|\nu_{\beta k}\rangle. \tag{3}$$

This equation shows that the original neutrino flavor becomes a superposition of all the flavors after travelling for a certain distance L.

The neutrino oscillation probability is derived from Schrodinger's equation, and is given by

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | v_{alpha}(L) \rangle|^2 \tag{4}$$

$$= |\sum_k U_{\alpha k} e^{-im_k L/2E} U^*_{\beta k}|^2 \tag{5}$$

which expands out to

$$P_{v_{alpha} \to v_\beta}(L) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2Re \sum_{k>j} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} e^{-i\Delta m^2_{kj} L/2E}. \tag{6}$$

2.1 The 2D Case

For the two flavour case, the mixing matrix simplifies to a two dimensional matrix with only one parameter, the mixing angle (θ).

$$U = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \tag{7}$$

Using equation 6, the probability of a neutrino oscillating from one flavour into another becomes:

$$P_{(\nu_\alpha \to v_\beta)} = (sin^2\theta cos^2\theta + cos^2\theta sin^2\theta) + 2(cos\theta)(-sin\theta)(sin\theta)(cos\theta)cos(\Delta m^2 L/(2E)) \tag{8}$$

$$= 2cos^2\theta sin^2\theta(1 - cos(\Delta m^2 L/(2E))) \tag{9}$$

$$= 2sin^2(2\theta)sin^2(\Delta m^2 L/(4E)) \tag{10}$$

In this equation, the factor $2\sin^2 2\theta$ determines the amplitude of the oscillation from one flavour to another, and Δm^2 for a given energy and angle determines the oscillation period. The neutrino oscillation will be at a maximum when $\theta = \pi/4$, meaning that at some point all of the initial flavour of neutrino will change into the other. When θ is equal to zero, no oscillation is possible and so the flavour and mass states are identical. Also, the mass squared difference is the difference in squared masses of the two different states: $\Delta m^2 = m_1^2 - m_2^2$. Therefore, it can be seen that at least one of the neutrino masses must have a non zero mass and that the mass states are not equal for neutrino oscillations to occur. Since neutrino oscillation experiments measure the probability of oscillation, they can only tell the difference between the masses and can not investigate the mass hierarchy and measure the absolute masses of neutrinos on their own. The parameters L and E, the distance from the source to the detector and the energy of the neutrino respectively, can be adjusted according to which parameter is being investigated.

2.2 The 3D Case

For the three flavour case, the mixing matrix has six possible parameters. Three mixing angles θ_{12}, θ_{13}, θ_{23}, one complex phase δ and the Majorana constants α_1 and α_2. The mixing matrix becomes

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_1 2 & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\alpha_1/2} & 0 & 0 \\ 0 & e^{-i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (11)$$

This matrix can be expanded out to equal

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{-i\alpha_1/2} & 0 & 0 \\ 0 & e^{-i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (12)$$

where $c_{ij} = cos\theta_{ij}$ and $s_{ij} = sin\theta_{ij}$. The α terms are only non-zero if neutrinos are Majorana particles and the δ term is only non-zero if there is CP violation.

2.3 CP Violation

The universe is composed of matter, and anti-matter is only seen in short lived high energy collisions. If there were equal amounts of matter and anti-mattery, they would annihilate each other. CP symmetry is a way to explain how the universe arrived at the unbalanced matter state of matter it has today. The Charge symmetry is the symmetry between a particle and an anti-particle. Applying Charge symmetry to an electron for example $C(e^-) \rightarrow e^+$, turns it into a positron (an electron of the opposite charge). This symmetry operation can be applied to a whole particle reaction, and if charge symmetry is conserved, then the two reactions will have the same rates, and behaviours. The Parity symmetry is the symmetry between a reaction and the reaction as seen in a mirror. The Parity symmetry flips the signs of particle spins. The parity symmetry is conserved if the reaction seen normally, and the one seen in a mirror happen in the same way. Weak decays were found to violate parity symmetry. The decay of a right handed muon to an electron and neutrinos was found to have a preferred decay direction, which is different if viewed in a mirror. CP symmetry is the symmetry, where both parity and charge symmetry are flipped. CP symmetry is conserved in weak interactions, as far as we know from current experiments. If a particle has CP violation, then according to CPT invariance, the symmetry must be preserved by having a time reversal symmetry as well.

The only current evidence of CP violation is from the quark mixing matrix and have been observed in neutral kaon and β decays. The CKM matrix however, is almost diagonal and therefore these CP violation events are rare. The MNS matrix for neutrino oscillation has larger angles so this area of study has a greater chance of measuring the CP phase. Now that the last angle, θ_{13}, for the the neutrino mixing matrix has been found to be sufficiently large, that is exactly what long base line experiments are now gearing up for. Once the CP phase for the MNS matrix is measured, it can possibly lead to new understandings about the beginning of the universe. One such theory is called leptonogenesis.

2.4 Mass Hierarchy

It is not known whether neutrinos have a normal mass hierarchy or if it is inverted. It is not possible to tell just from looking at neutrino oscillations but can be discovered by looking at the matter effect. Taking a look at Figure 1, the mass splitting between two of the states is much smaller than to the other. Therefore $\Delta_{31}^2 \approx \Delta_{32}^2$ and thus the two remaining mass splittings can be measured with minimal interference from each other. Experimental setups using long base paths and high energies are well suited to explore this issue. This is because, the θ_{13} mixing angle is more sensitive to electron-neutrino appearace for the normal mass hierarchy due to the higher density of electrons.

2.5 Dirac versus Majorana particles

A Dirac particle has an antiparticle while a Marjorana particle is its own antiparticle. In the standard model of particle physics neutrinos are considered to be Dirac particles, but there is no evidence to rule out the possibility that they could be Majorana. The Marjorana phases in the mixing matrix unfortunately cancel out so studying simply neutrino oscillations can offer no conclusive results.

An experiment that could differentiate between Marjorana and Dirac particles is the double beta decay experiments. Neutrino-less double-beta decay is a very rare phenomenon which involves two neutrons decaying into two protons and two electrons while emitting two neutrinos as shown in FIG.2. This decay is only possible if neutrinos are their own anti-particle which can also help quantify the neutrino mass.

If neutrinos are in fact Majorana particles, then their lepton number will not be conserved and neutrinos will have left-handed helicity while their anti-particle will be right-handed.

3 Neutrinos travelling through matter

Since neutrinos interact weakly, they propagate differently when going through matter than when they go through vacuum. A potential is formed due to the coherent forward elastic scattering between the neutrinos and the particles in the medium. This modifies neutrino mixing as the potential acts as an index of refraction. Two different kinds of potentials are produced through the weak force: neutrinos can go through the Z exchange to produce a neutral potential, V_{NC}, while only the electron neutrino can go through an exchange with the W boson and produce a charged potential, V_{CC}. Incoherent scattering is assumed to be negligible except in special cases such as in neutron stars. It requires a high density for matter to be considered opaque to neutrinos. For example, the mean free path for solar neutrinos is of the order of 0.1 light years while the mean free path for a neutron star is of the order of 1 km. So for a neutron star of 1 km in diameter, incoherent scattering must be taken into account.

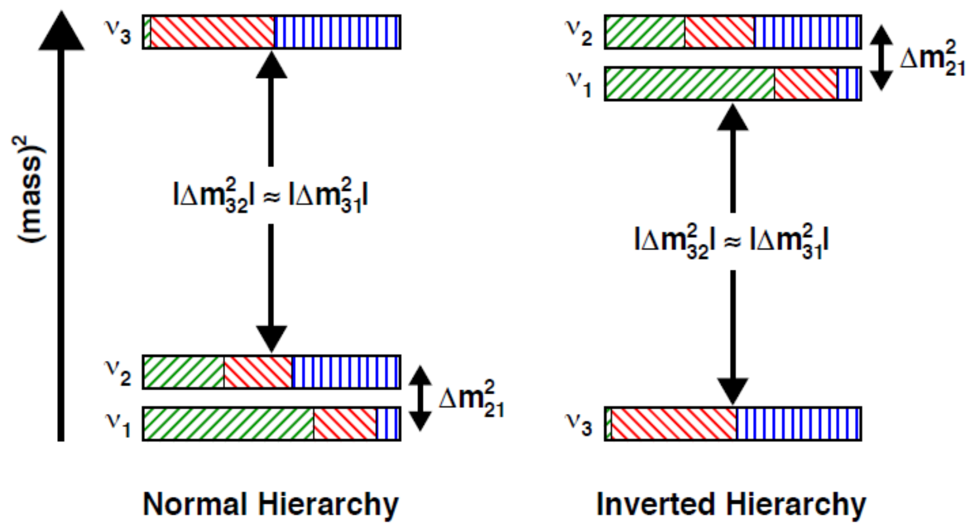

Figure 1: To the left is the normal neutrino mass hierarchy and to the right is the inverted hierarchy where green indicates ν_e, red indicates ν_μ and blue indicates ν_τ.

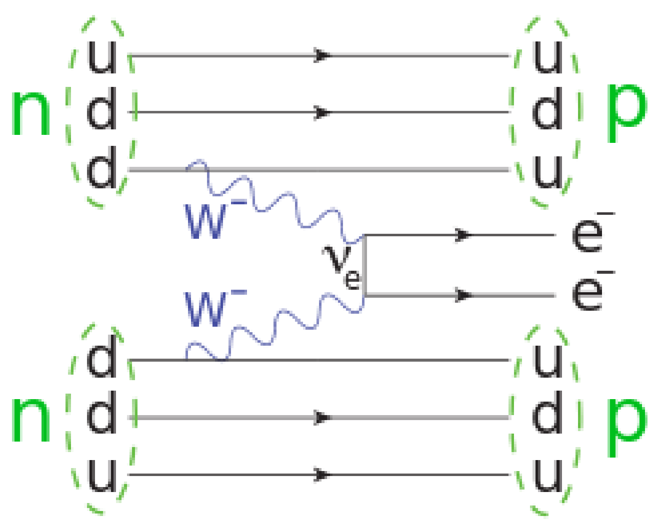

Figure 2: Feynmann diagram of zero neutrino double beta decay ($0\nu\beta\beta$) [5].

The effective neutrino Hamiltonian has the potential

$$V_\alpha = V_{CC}\delta_{\alpha e} + V_{NC} = \sqrt{2}G_F(N_e\delta_{\alpha e} - N_n/2) \tag{13}$$

For the neutral current potential, only neutrons contribute since it is implied that the protons and electrons cancel out[3]. Since V_{NC} is the same for all neutrino flavours it does not change the oscillation probabilities.

When assuming an non-vacuum state, the initial Hamiltonian in vacuum is modified to account for the potential

$$H = H_0 + H_1 \tag{14}$$

such that $H_0|\nu_k\rangle = E_k|\nu$ and $H_1|\nu_\alpha\rangle = V_\alpha|\nu_\alpha\rangle$ where $E_k = \sqrt{(}p^2 + m_k^2)$. The time evolution equation for the flavor transition from α to β is given by

$$i\frac{d}{dx}\Psi_{\alpha\beta}(x) = (p + m_1^2/2E + V_{NC})\Psi_{\alpha\beta}(x) + \tag{15}$$

$$\sum_n(\sum_k U_{\beta k}\frac{\delta m_{k1}^2}{2E}U_{nl}^* + \delta_{\beta e}\delta_{ne}V_{CC})\Psi_{\alpha n}(x)), \tag{16}$$

given that $E_k = E + m_k^2/2E$, $p \simeq E$ and $t \simeq x$ for ultrarelativistic neutrinos. The first term of the above equation can be eliminated by a phase shift and has no effect on flavor transitions. So the effective Hamiltonian in the flavor basis can be given by

$$H_f = 1/2E(\delta m_{31}^2 U_{\alpha 3}U_{\beta 3}^* + \delta m_{21}^2 U_{\alpha 2}U_{\beta 2}^* + A_{CC}\delta_{\alpha e}\delta_{\beta e}) \tag{17}$$

where $A_{CC} = 2\sqrt{2}EG_fN_e$ and $\delta m_{jk}^2 = m_j^2 - m_k^2$.

3.1 MSW Effect

There is a resonance effect when the effective mixing angle is at $\pi/4$ which allows for a substantial mixing of neutrinos even when the vacuum mixing angle is small. The effective Hamiltonian for a two flavor basis is

$$H_F = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 cos2\theta + A_{CC} & \Delta m^2 sin2\theta \\ \Delta m^2 sin2\theta & \Delta m^2 cos2\theta - A_{CC} \end{pmatrix} \tag{18}$$

When diagonalized by the following orthogonal transformation, one can find the effective Hamiltonian in the mass basis of matter.

$$H_M = U_M^T H_F U_M = \frac{1}{4E} diag(-\Delta m_M^2, \Delta m_M^2) \tag{19}$$

Where

$$\Delta m_M^2 = \sqrt{(\Delta m^2 cos2\theta - A_{CC})^2 + (\Delta m^2 sin2\theta)^2} \tag{20}$$

is the effective squared-mass difference. This mass difference versus matter density is shown in FIG. 3.

The effective mixing angle in matter, θ_M can be solved to be

$$\tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{A_{CC}}{\Delta m^2 cos2\theta}}. \tag{21}$$

The mixing angle versus matter density is shown in FIG. 4.

The resonance that Mikheev and Smirnov discovered happened when A_{CC} becomes equal to $\Delta m^2 \cos 2\theta$. The mixing angle is maximal at $\pi/4$ and if the resonance region is wide enough, there can be a total transition between two flavors. In matter, A_{CC} has to be positive so θ must be less than $\pi/4$ for neutrinos and greater than $\pi/4$. This is different than neutrino

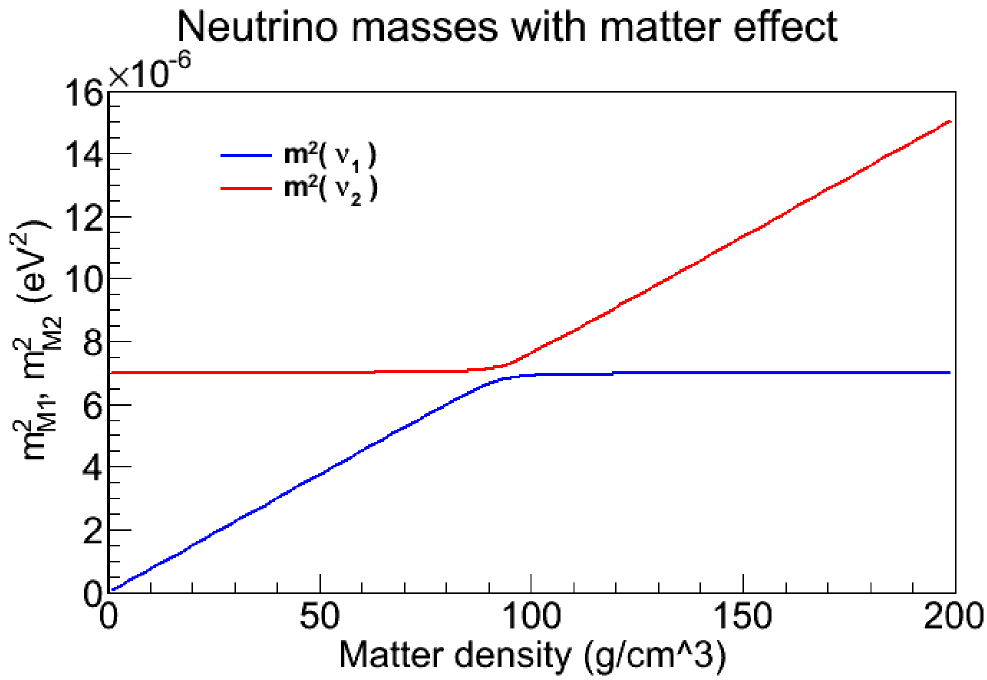

Figure 3: MSW resonance seen in effective mass-squared as a function of electron density.

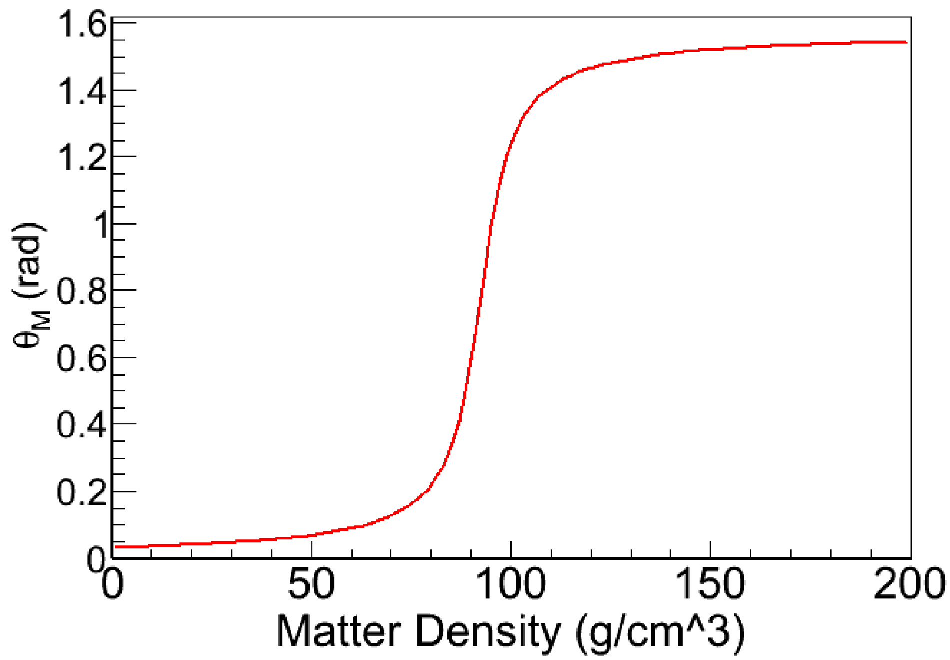

Figure 4: MSW resonance seen in effective mixing angle as a function of electron density.

oscillations in a vacuum since the probability is symmetric when θ goes to $= -\theta$.

Figures 3 and 4 show how the density of matter can drastically affect neutrino oscillations. When the effective mixing angle is greater than 45 degrees and the density is large, there can be an almost complete production of the other neutrino even with a small mixing angle. For the MSW effect to occur, adiabacity is required since the neutrinos may jump from flavor to flavor and the gap between energy eigenstates at resonance will be too small.

4 T2K

There are many current experiments researching neutrinos. The Tokai to Kamioka (T2K) experiment is a long base line neutrino experiment in Japan. The experiment starts off in the J-PARC facility, which produces a mainly ν_μ beam, which is then measured using a near detector 280 m from production, and 295 km away at the far detecor (Super-Kamiokande). Figure 5 shows the relative locations of the J-PARC neutrino beam, the near detector, and the far detector.

4.1 T2K neutrino beam

The beam is emitted by the accelerator into a 30 GeV synchrotron where it collides with a graphite target at high temperatures, producing pions and kaons. These particles are focused by a magnetic field into a pion decay volume and decay into mainly muon neutrinos.The relatively small admixture of other flavours of neutrinos in the beam can be explained by the decay sequence $\pi^+ \rightarrow \mu^+\nu_\mu$, and subsequent decay of $\nu^+ \rightarrow e^+ \nu ebar \nu_\mu$. Assuming that they both have the same amount of time to decay, then percentage of ν_e ends up being the inverse of the ratio of their lifetimes:

$$\frac{N_{\nu_e}}{N_{\nu_\mu}} \approx 0.01 \tag{22}$$

which is almost negligible but should be accounted for to insure accuracy in the measurements in the Far Detector. Improvements in particle identification in the Near Detector can help determine how much of the beam is ν_μ and ν_e.

The T2K neutrino beam is an off-axis beam, to pick out an energy spectrum that is peaked at the oscillatiom maximum for atmospheric neutrinos. The effect of the off axis beam is shown in FIG. 6. The reason why the off-axis beam is less broad in energy than the on-axis beam can be derived from the conservation of momentum of pion decay, $\pi \to \mu\nu$. Considering the pion and the neutrino in particular, the energy-momentum conservation can be written as

$$\mu = \pi - \nu \tag{23}$$

If we take the z axis to be pion's direction in the lab frame, the 4-vectors for π and ν can be written as

$$\pi = (m_\pi, 0, 0, 0) \tag{24}$$

$$\nu = (E_\nu^*, E_\nu^* sin\theta^*, 0, E_\nu^* cos\theta^*). \tag{25}$$

By squaring the conservation equation and plugging in $(\pi \cdot \nu) = m_\pi E_\nu^* *$, one can solve for E_ν^*:

$$E_\nu^* = \frac{m_\pi^2 - m_\nu^2}{2m_\pi} = 30MeV. \tag{26}$$

Looking at the neutrino 4-vector, we can give it a Lorentz boost $\gamma_\pi = E_{pi}/m_\pi$ and then use that to solve fortan θ. Assuming $E_\pi \gg m_\pi$, $\gamma_\pi \gg 1$ and $\beta_\pi \approx 1$, then

$$tan\theta = \frac{E_\nu^* sin\theta^*}{\gamma_\pi E_\nu^*(1 + cos\theta^*)} \approx \frac{E_\nu^* sin\theta^*}{E_\nu}. \tag{27}$$

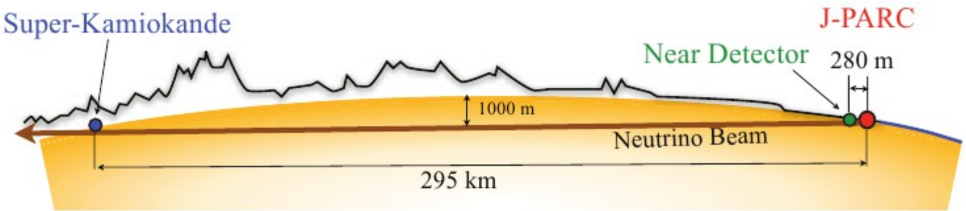

Figure 5: Cartoon of Tokai to Kamioka experiment showing the neutrino beam, near detectors and far detector.

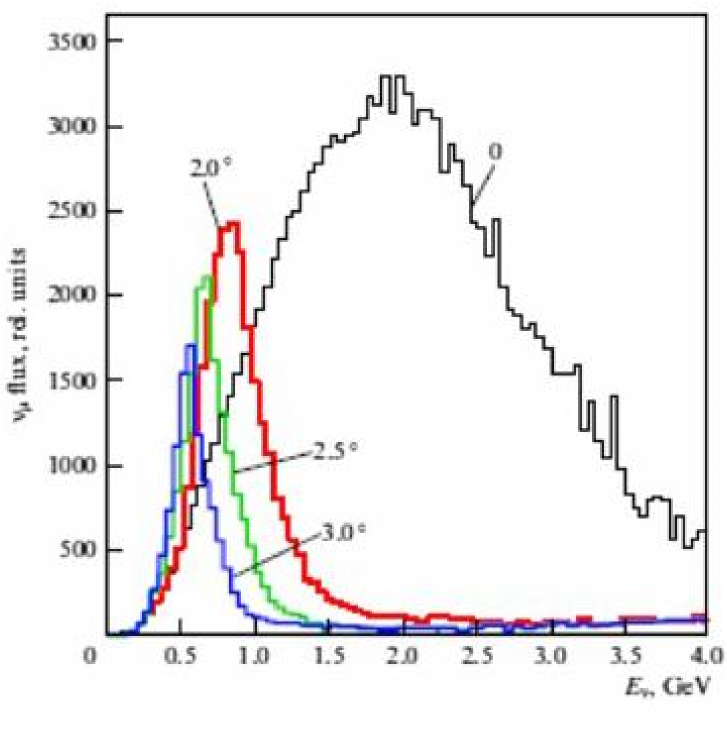

Figure 6: Muon neutrino energy spectrum for on-axis, and several off-axis angles[1].

Due to the preceding equation, there has to be some θ_{max} since $\sin\theta$ cannot be greater than one. Therefore, $\theta_{max} = \frac{30MeV}{E_\nu}$.

Now, it is possible to derive the neutrino spectrum given that the energy spectrum is $dN/dE_\pi \propto (E_p - E_\pi)^5$. Since the spectrum in the pion rest frame is isotropic, the following approximation can be made

$$\frac{d^2N}{dE_\nu d\Omega} \propto \frac{d^2N}{dE_\nu d\cos\theta} \tag{28}$$

as well as this transformation

$$\frac{d^2N}{dE_\pi d\cos\theta^*} J(E_\pi, cos\theta^*; E_\nu, cos\theta) \propto (E_p - E_\pi)^5 J. \tag{29}$$

Using relations in equations 25 and 27, the Jacobian can be solved to be approximately $J \propto \frac{E_\pi E_\nu}{E_{nu}^{2*} cos\theta^*}$ and then the neutrino spectrum can be written as

$$\frac{d^2N}{dE_\pi d\cos\theta} \propto (E_p - E_\pi)^5 \frac{E_\pi E_\nu}{cos\theta^*}, \tag{30}$$

meaning that is possible for the neutrino flux to be higher for nonzero values of θ since the denominator in the previous equation can become zero.

4.2 The near detectors

Further downstream are T2Ks two near detectors. INGRID FIG. 8 and ND280 FIG. 7 both monitor and profile any escaping high momentum tracks through their Electromagnetic Calorimeter. INGRID is a large volume neutrino beam monitor comprised of several modules arranged in a plus shape to see both the horizontal and vertical profile of the neutrino beam. The ND280 detector is inside of an 0.2 T magnetic field, and has several sub-detectors. The inner-most part of ND280 is called the tracker region, and consists of a large mass $\pi 0$

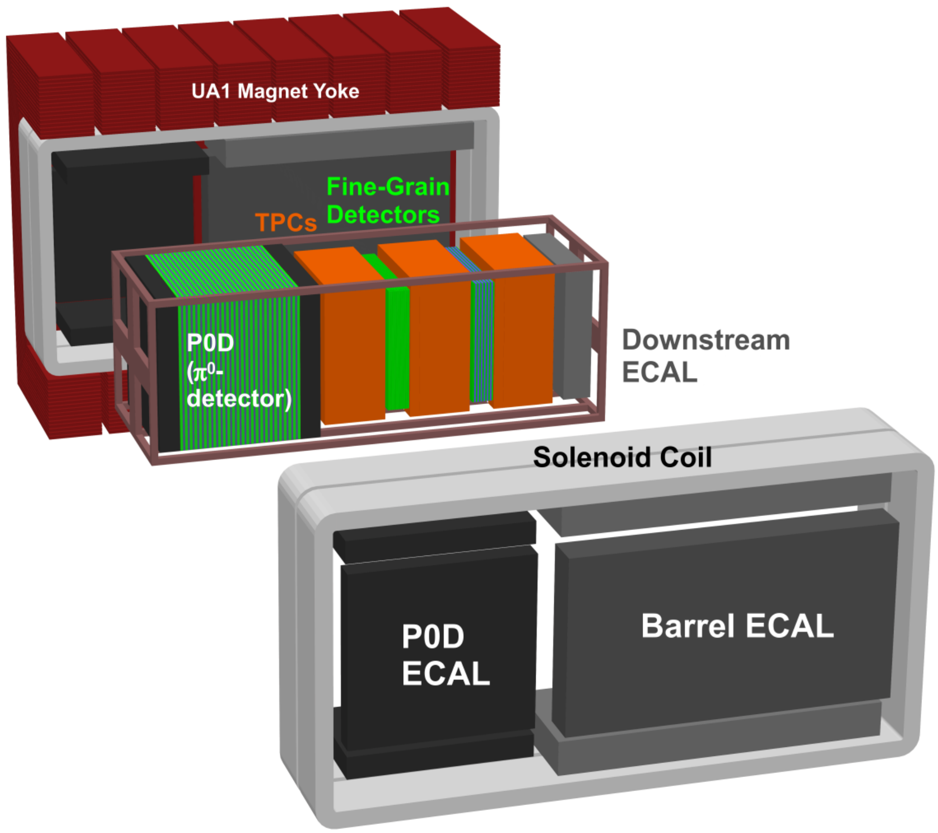

Figure 7: The Near Detector ND280

detector called the P0D, and two large scintillator based Fine Grained Detectors (FGD)s, sandwiched between three Time Projection Chambers (TPC)s. The FGDs provide the target mass for the neutrinos to interact in, and the TPCs provide good momentum resolution and electron-muon separation. An electromagnetic calorimeter surrounds the P0D, TPCs and FGDs. Also, layers of scintillator called the side muon range detector (SMRD) are placed between the iron of the magnet yokes.

4.3 Super-Kamiokande

The Far Detector which contains Super-Kamiokande (SK), is situated at the end of the beam pathway and uses a massive Cherenkov detector FIG. 9 to study neutrino interactions. Most neutrinos pass through the detector without interacting, but those which do interact wiith the water and emit a charged particle. When the charged particle travels faster than the speed of light in water, the displaced water particles emit a light cone known as Cerenkov radiation. The detector itself is lined with ten thousand photo-multipliers that can detect the radiation and differentiate between muons and electrons.

5 Investigating T2K potential for new physics discovery

Neutrino experiments around the world have been meausring neutrino oscillation parameters to increasingly higher degrees of accuracy FIG. 10. T2K has been a large contributor, especially towards the measurement of $theta_{13}$. Now that the mixing angles and mass differences have been acurately meausred, the new goals of T2K include determining looking for hints of any CP phase and clarifying the mass hierarchy. An expansion of T2K could

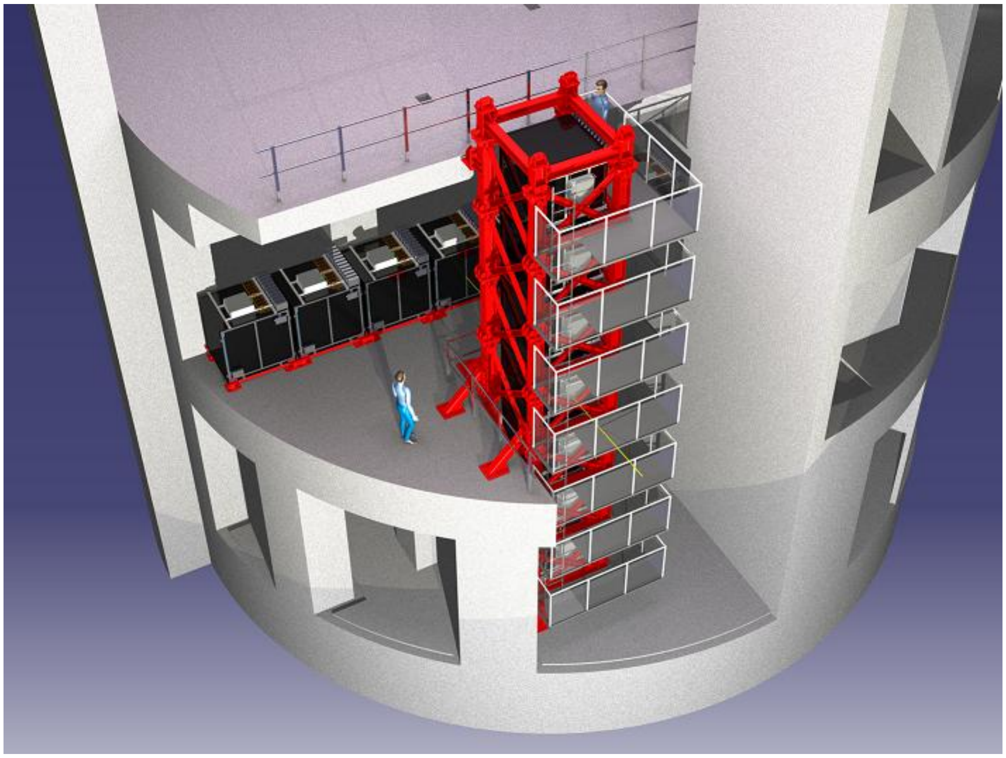

Figure 8: The Near Detector Ingrid

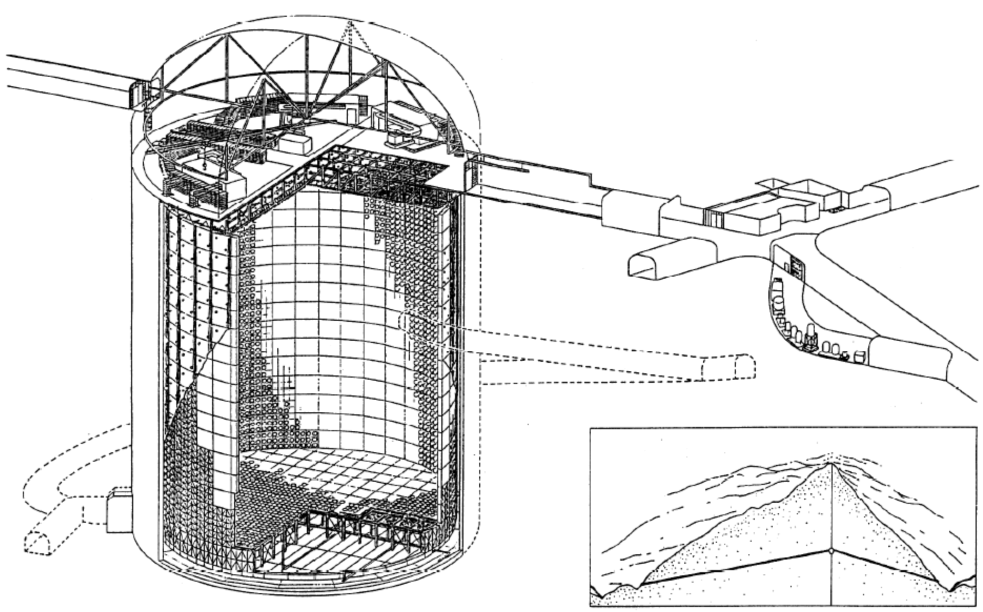

Figure 9: This is a photograph of Super-Kamiokande which is situated 1000 metres beneath the surface

help disentangle the matter effect and a comparison between the oscillation probabilities for neutrino and anti-neutrino beams could lead to the discovery of CP-violation.

This thesis will look at the effects of CP-violation, matter-effect, and mass hierarchy at T2K, and possible extensions to T2K.

5.1 Neutrino Oscillation Equations for Constant Density Matter

For measuring CP violation effects, it is useful to introduce the following notation when assuming constant density matter:

$$\Delta = |\delta m_{31}^2| L/4E_\nu = 1.27|\delta m_{31}^2/eV^2|(L/km)/(E_\nu|/GeV) \quad (31)$$

$$\hat{A} = |A/\delta m^2 31| \quad (32)$$

$$\alpha = |\delta_{21}^2|. \quad (33)$$

Using the notation, the oscillation probabilities are:

$$P(\nu_\mu \to \nu_e) = x^2 f^2 + 2xyfg(\cos\delta\cos\Delta - \sin\delta\sin\Delta) + y^2 g^2 \quad (34)$$

$$\bar{P}(\bar{\nu}_\mu \to \bar{\nu}_e) = x^2 \bar{f}^2 + 2xy\bar{f}fg(\cos\delta\cos\Delta + \sin\delta\sin\Delta) + y^2 g^2 \quad (35)$$

where

$$x = \sin\theta_{23}\sin 2\theta_{13}, \quad (36)$$

$$y = \alpha\cos\theta_{23}\sin 2\theta_{12}, \quad (37)$$

$$f, \bar{f} = \sin((1 \mp \hat{A})\Delta)/(1 \mp \hat{A}), \quad (38)$$

$$g = \sin(\hat{A}\Delta)/\hat{A}. \quad (39)$$

for $\delta m_{31}^2 > 0$ and $\delta m_{31}^2 > 0$. To find the probabilities for anitneutrinos, $\delta m_{31}^2 < 0$, the transformation due to matter effects, $f \rightarrow \hat{f}$ and the negation of $\hat{A}$, y, g and Δ must occur.

$$P(\nu_\mu \rightarrow \nu_e) = x^2 \bar{f}^2 - 2xy\bar{f}g(\cos\delta\cos\Delta + \sin\delta\sin\Delta) + y^2 g^2 \tag{40}$$

$$\bar{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = x^2 f^2 - 2xyfg(\cos\delta\cos\Delta - \sin\delta\sin\Delta) + y^2 g^2 \tag{41}$$

From these probability equations, it is possible to see that when g = 0, the CP phase and y^2 term both vanish. This implies that when the distance of the base path length is close to the oscillation length due to matter, the CP phase becomes suppressed and θ_{12} and δm_{21}^2 cancel out meaning that this would be a good set up for measuring θ_{13}.

5.2 CP Violation effect at T2K

CP violation has not been meausred at T2K but its effect on neutrinio oscillation patterns can be very drastic FIG. 11. To measure δ, a longer baseline experiment than T2K is required but T2K's high sensitivity to θ_{13} could still be useful since in the MNS matrix, these two terms are linked together.

5.3 Matter effect with T2K

The matter effect grows more pronounced at longer distances. T2K's beam path length is only 295 km and it can be seen from FIG. 12 that the matter effect is almost neglibeable. This means that T2K has an ideal base path length to measure CP violation with less interference from the matter effect.

parameter	best fit	1σ range	2σ range	3σ range
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.62	7.43–7.81	7.27–8.01	7.12–8.20
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\text{eV}^2]$	2.55 2.43	$2.46 - 2.61$ $2.37 - 2.50$	$2.38 - 2.68$ $2.29 - 2.58$	$2.31 - 2.74$ $2.21 - 2.64$
$\sin^2\theta_{12}$	0.320	0.303–0.336	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	0.613 (0.427)[a] 0.600	0.400-0.461 & 0.573–0.635 0.569–0.626	0.38–0.66 0.39–0.65	0.36–0.68 0.37–0.67
$\sin^2\theta_{13}$	0.0246 0.0250	0.0218–0.0275 0.0223–0.0276	0.019–0.030 0.020–0.030	0.017–0.033
δ	0.80π -0.03π	$0 - 2\pi$	$0 - 2\pi$	$0 - 2\pi$

Figure 10: Current Values of the mixing angles and mass differences for neutrino parameters. When two values are given, the top one corresponds to the normal hierarchy and the bottom to the inverted hierarchy. [6]

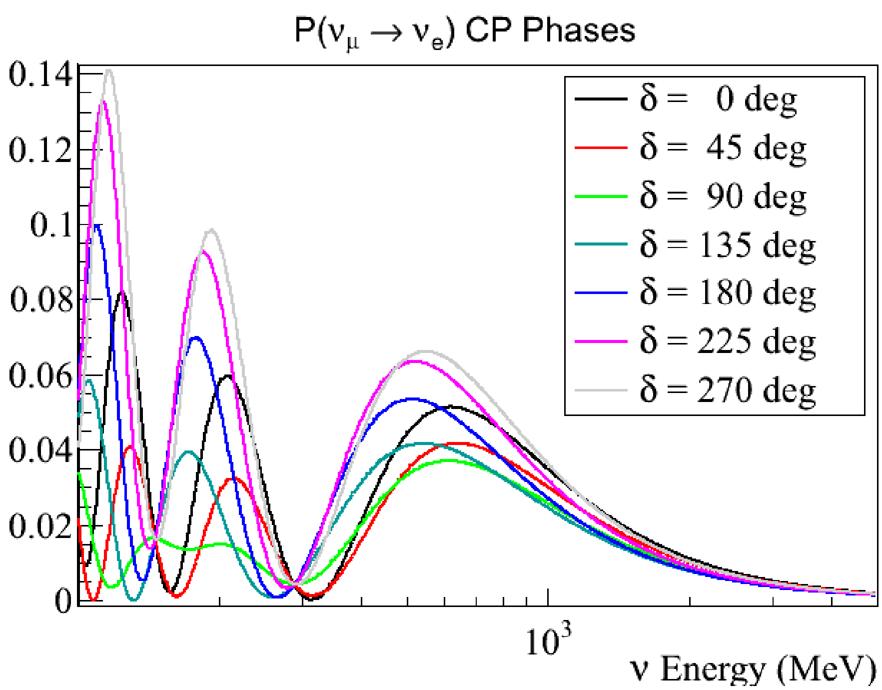

Figure 11: Electron-neutrino appearance probability as a function of energy for several different CP-Phases, where the other oscillation parameters reamin fixed at their current values, and the matter density is 2.6 g/cm^3.

5.4 Matter effect at different distances

The matter effect is negligible at T2K but at longer base paths, the matter effect could affect neutrino oscillations and would have to be accounted for. There has been talk of installing detectors at different base paths away from T2K that would make an ideal setup to quantify the CP phase and the mass hierarchy. As seen FIG. 13, the probability of neutrino oscillations differs greatly by base path and can be used to discover new properties.

Also, long base line experiments can be useful to detect CP violation by measuring both neutrino and anti-neutrino oscillations and comparing the two. When plotting the two probabilities together for a certain base path, one can see how well they agree with each other. Longer base paths in conjuction with higher beam energy are ideal to explore CP violation. Two possible sites include the Oki Island and one in Seoul, South Korea. Oki Island lies 693 km away from Tokai along the base path to Super K and because of the longer base path, the T2K experiment would be much more sensitive to the difference between the normal and inverted hierarchy. The one in Korea can also be used as well. Once the hierarchy has been established, the focus will shift to measuring the CP phase.

5.5 Effect of Matter Hierarchy seen at T2K

T2K can be sensitive to the matter hierarchy. Depending whether it is inverted or not, the neutrino oscillation probabilities will shift accordingly FIG. 14. Although the effect can be said to be slight, it is import to determine the mass hierarchy so that increased accuracy in measurements can be achieved.

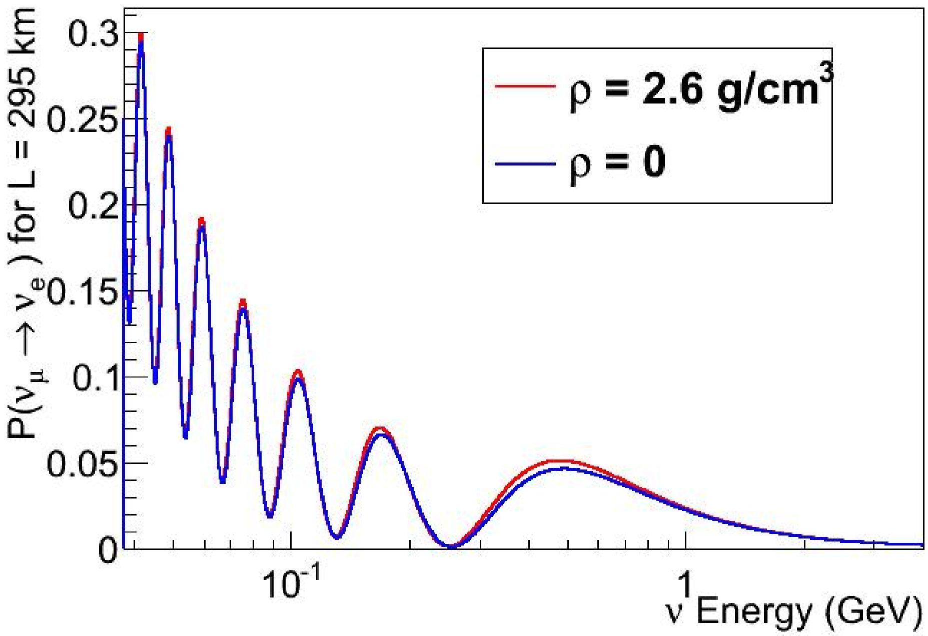

Figure 12: Electron-neutrino appearance probability as a function of energy with and without the matter-effect shows that this signal is not senstive to the matter effect.

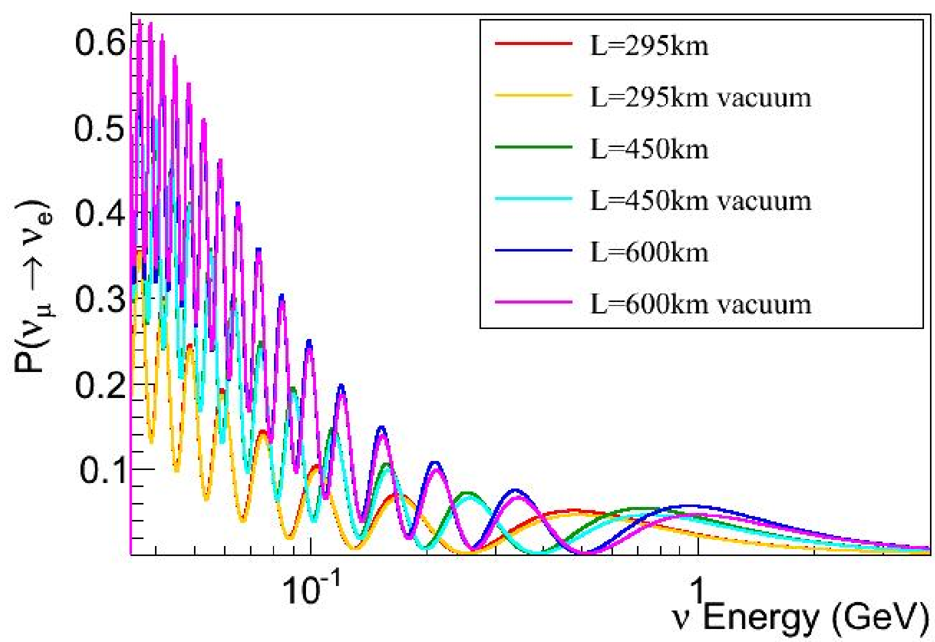

Figure 13: electron-neutrino appearance probability as a function of energy for several different base path lengths, both with and without matter effects.

5.6 Disentangling CP effect

In order to see how to get the CP phase in the presence of matter effects, and an uncertain matter hierarchy we investigate the oscillation probablility for neutrino and anti-neutrino cases, when most of the mixing parameters are already well known. For each value of the CP-phase, we plot oscillation probability for the anti-neutrino case, versus the neutrino case. As we change the CP-phase, this plots out an ellipse in $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ vs $P(\nu_{mu} \rightarrow \nu_e)$. We also plot this for both hierarchies overlaid, and we can see that the effect is different for different neutrino energies, and for different neutrino baselines.

For the following graphs, neutrino oscillation probabilities vs anti-neutrino oscillation probabilities have been plotted. Figures 15 and 16 show CP-phase ellipses, which are the anti-neutrino oscillation probability versus neutrino oscillation probability as δ is swept through 0 to 2π. Figure 15 shows the CP-phase ellipse for a base path length of 295 km at several energies, and FIG. 16 shows the CP-phase ellipse for a base path length of 1290 km. When δ is not a multiple of $\pi/2$, the orbit for θ_{13} is an elipse. When $\delta = (n - \frac{1}{2})\pi$, then the ellipse collapse into a line and CP violation can be measured directly by comparing the different event rates. If $\delta = n\pi$, the ellipse also collapse into a line, and the CP violation can be measured indirectly by parametization.

6 Conclusion

Neutrino's have been experimentally determined to oscillate in matter which can be calculated by treating neutrinos oscillations as plane waves. When considering all three flavours, there are three mixing angle parameters and one CP phase. All three mixing angles have now been measured, most recently θ_{13} and now experimenters are focusing on determining the mass hierarchy, exploring the matter effect and looking for an evidence of cp violation

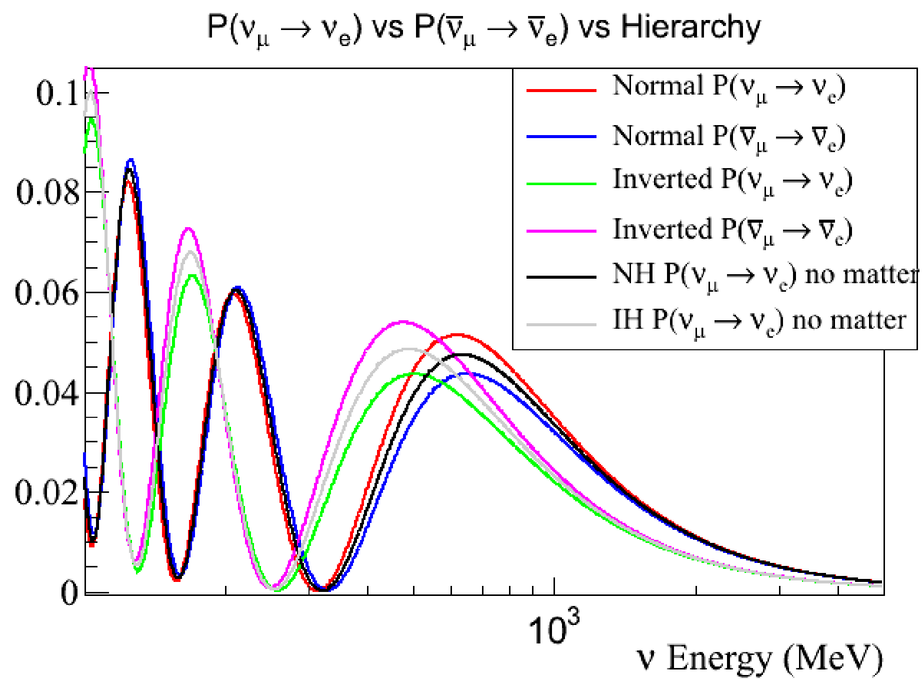

Figure 14: Electron-neutrino appearance probability as a function of energy to compare normal and inverted hierarchies, with and without matter effect..

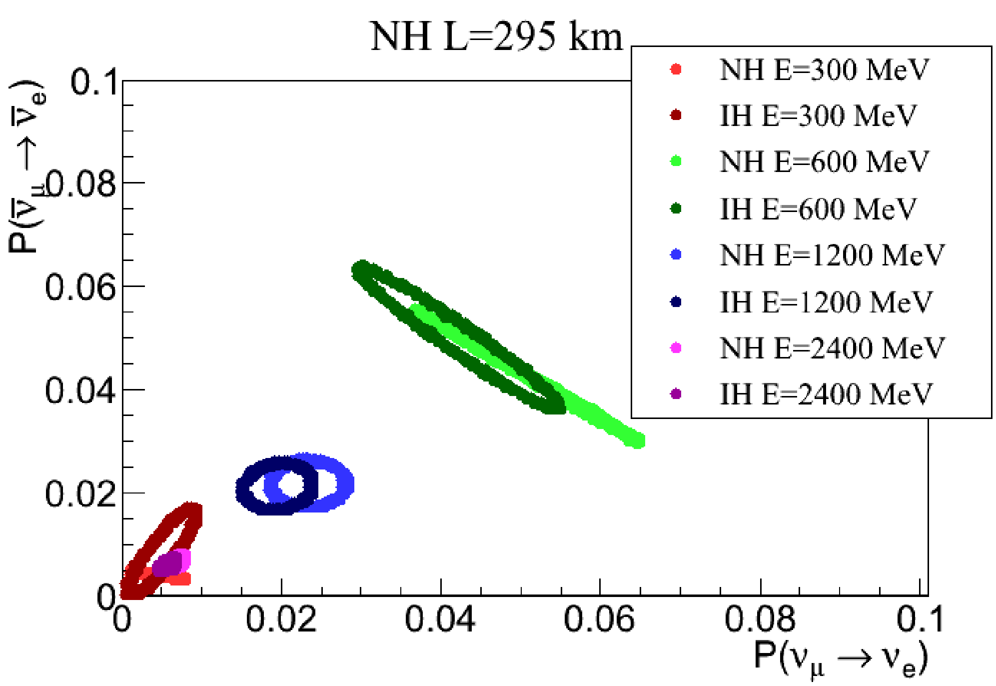

Figure 15: CP-Phase ellipses for T2K's base path length. Since the normal and inverted hierarchy ellipses overlap, disentangling changes in CP-phase from the matter hierarchy are difficult.

in neutrino oscillation. This will require upgrading equipment or modifying experimental setups such as the proposed expansion at T2K.

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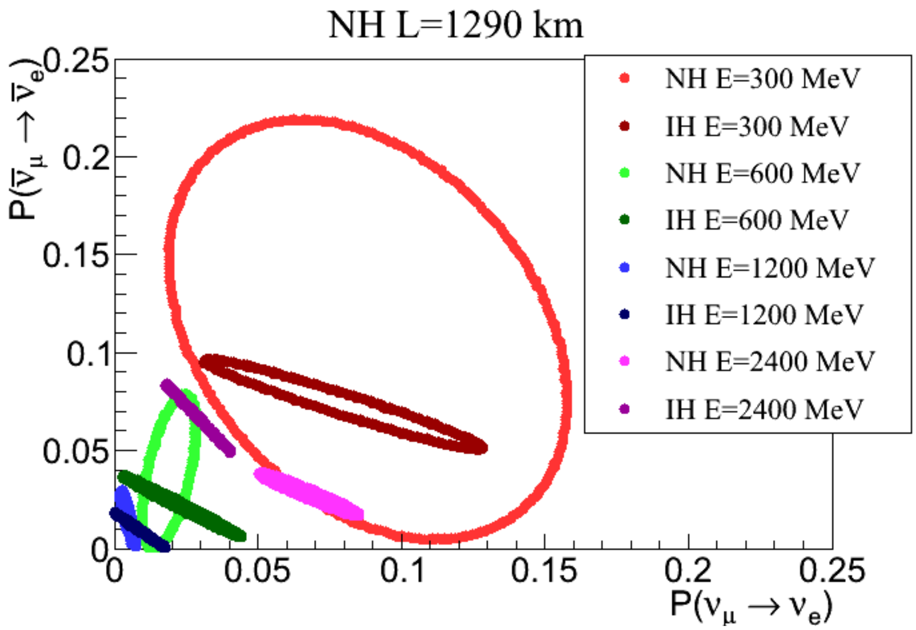

Figure 16: CP-Phase ellipses for a base path length of 1290 km. In this case the normal and inverted hierarchy ellipses do not overlap as much, making it easier to disentangle changes in CP-phase from the matter hierarchy.