



Bachelor thesis

# HE-II CRYOSTAT STUDIES FOR A NEW ULTRA COLD NEUTRON SOURCE

Ultra Cold Advanced Neutron source (TUCAN) Vancouver

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# Abstract

In order to explain how the universe came into being, scientist want to explain the huge mater-antimatter imbalance in the universe. The solution should be found with the help of neutrons. However to be able to examine them first they need to be produced. So this thesis looks at the production process of neutrons. Since neutrons are high energetic when they are produced, a cryostat is necessary to cool them down to temperatures where neutrons can be prated. To obtain the required cooling a helium cryostat with superfluid helium is used to cool the neutrons down. Therefore, a closer look on is taken on superfluid helium and its remarkable properties like superflow and its unique heat transfer properties are shown. After that, the two main helium systems of the outlet are analyzed with a discussion about every component of the helium cryostat referring its required and supplied cooling power. It will be seen that some of the heat exchangers can be neglected because they provide almost no cooling. Some of the heat changers can't be removed but they can designed in a easier way, what simplifies the construction of these specific heat exchangers. Since we obtained the performance of the pump for an ideal case, where the pressure isn't changing through the outlet, we will notice that this influence can cause a pumping volume to up to 217 000  $\frac{\text{m}^3}{\text{h}}$ . That case would require an enormous pump and very high costs. Therefore, to avoid this case the pressure drop is examined for the straight and bended sections of the outlet. At the end, it is found out that an increased outlet diameter at the end of the construction can prevent almost the whole pressure drop. At last the Kapitza conductance at a liquid-solid interface is discussed in terms of its two existing theories. These theories are not a good assumption of the conductance so we will look closer at the whole dependencies and find a strong dependence on the surface morphology at the crossing. Furthermore a similar heat examiner is examined referring its temperature increase by Kapitza conductance and we will see that the obtained values are in agreement with our earlier obtained equations for the Kapitza conductance. Because of that proven good agreement, we apply the knowledge to older measurements at the vertical heat exchanger and we will see that these measurements are too high. This could be caused by bad calibrated temperature sensors or a changed boiling regime. Finally the obtained knowledge is applied to the new design of the heat exchanger, what will conclude in an total temperature increase of 0.15 K by Kapitza conductance.

# Contents

1	Intr	roduction	1		
	1.1	Motivation	1		
	1.2	Neutron properties and production	1		
	1.3	UCN production	3		
	1.4	UCN loss	4		
<b>2</b>	Sup	erfluid helium	8		
	2.1	Helium phases	8		
	2.2	Properties of superfluid helium	8		
3	Cryostat design 14				
	3.1	Comparison of helium isotopes	14		
	3.2	Basic physical principles	15		
	3.3	The two main helium systems	23		
		IP Helium system	23		
		$^{3}$ He system $\ldots$	24		
	3.4	Calculations of the cryostat	28		
		Performance of the HEX	28		
		Importance of the HEX	34		
<b>4</b>	Pre	ssure Drop	40		
	4.1	Design of the <sup>3</sup> He outlet $\ldots$	40		
	4.2	Pressure drop in a straight pipe	40		
	4.3	Pressure drop in 90°-bendings	45		
	4.4	Basic dependencies of pressure drop	46		
	4.5	Effects of the pressure drop	48		
	4.6	Achieving a low pressure drop	49		
	4.7	Separation of the outlet	50		
<b>5</b>	Кар	bitza conductance	<b>52</b>		
	5.1	Surface morphology	57		
	5.2	Comparison of previous results	59		
	5.3	Heat exchanger of a dynamically polarized proton filter	59		
	5.4	Heat transfer regimes	61		
	5.5	Kapitza conductance at the new HEX design	65		
6	Cor	nclusion	69		

# List of Figures

$\frac{1}{2}$	<ul><li>[2] Composition of a proton and a neutron.</li><li>[3] An example of the charge displacement measurement. If the size of a neutron were</li></ul>	2
	equal to the size of the earth, then the measured upper limit for the charge displacement	
	would be 1 $\mu$ m	2
3	[36] The process of a proton (p) colliding with a tungsten nuclei, producing neutrons (n)	
4	and protons	4
	which enter the moderator system. The moderator system includes a layer of graphite	_
_	and lead, heavy water, liquid deuterium and d superfluid helium.	5
$\frac{5}{6}$	[36] The downscattering process of a neutron, from the cold to the ultra cold stage [37] The UCN production process, from the emitted high energy neutrons through the	5
	different moderator systems and the final cooling in superfluid helium. The scale on	
	the right hand side shows the temperature and velocity of the neutrons through the	
	different steps.	6
7	[38] The neutron energy loss rate from the phonons over an increasing temperature.	6
8	[12] The UCN guide, where the neutrons enter on the right side in the liquid Helium,	
	pass the heat exchanger, and finally leave through a guide filled with helium vapor	7
9	[6] Phase diagram of <sup>4</sup> He where the $\lambda$ -line describes the transition between He-I and He-II.	9
10	[34] The behavior of He-I and He-II in a beaker with a porous bottom.	10
11	[34] The construction of the second second viscosity experiment, in a) are the copper	
	cylinder and the wooden paddle wheel seen from the side and b) is the top view of the	
	whole experiment, including the four coils of the induction motor	10
12	[6] The experimental construction of the fountain effect. The liquid in the sphere gets	
	heated, consequently the liquid squirts out through the tube	11
13	[6] All beakers and baths are filled with He-II. Each part shows the movement of the	
	rolling film for different submerged depths and the various fillings of the beaker	12
14	[34] Schematic built of the second sound experiment.	14
15	Section a) depicts the heat of evaporation of ${}^{3}$ He and ${}^{4}$ He over the temperature. Part b)	
	illustrates the vapor pressure of <sup>3</sup> He and <sup>4</sup> He over the temperature as well.[14] $\ldots$	15
16	[6] The principle of staedy flow in a closed system.	16
17	The first part, a) shows he schematically function of a JT expansion, where the red dots	
	represent helium particles. b) depicts the icon of a JT value. $[6]$	17
18	The illustration shows the black inversion curve of ${}^{4}\text{He}$ and in green an example plot	
	for constant Enthalpy. At the inversion curve $\mu$ equals zero, the smaller part under the	
	Graph indicate $\mu$ smaller than zero and the remaining section stands for $\mu$ greater than	
	zero	18
19	A characteristic vaporization scheme of the cryostat. The Helium comes in at point	
	"0" and passes the JT valve. The majority of the Helium gets liquefied and a fraction	
	of it gets vaporized. Point "11" describes the liquefied Helium and "1v" the directly	
	vaporized Helium. During operation, the heat input, indicated at the bottom, converts	
	the liquid Helium $(11)$ to vaporous Helium $(21)$ . Both vapor streams are depicted by	
	point "3" together.	21
20	Parallel flow and counterflow are compared in terms of temperature exchange over time.	
	The blue medium is the coolant which cools the red flow. $[32]$	21

21	The illustration shows the principle of a counterflow heat exchanger. The Helium is at the beginning in the liquid phase, than it flows through the pipe until it gets cooled and liquefied by a Joule-Thomson valve. After that the liquid is contained in a vessel in	
	which it gets vaporised due to a heat input from underneath. The vapor gets pumped away through a pipe that is equipped with a part that is passed by the inlet pipe. In	
	this section is the hotter incoming gas cooled by the cooler escaping gas. [30]	22
22	The schematic cryostat with focus on the IP Helium flow. On its way to the UCN guide, the IP Helium is cooled down by the two <sup>4</sup> He baths and a JT valve. On the right hand	0.0
23	The illustration displays the schematic construction of the whole <sup>3</sup> He precooling system which is built out of the three refrigeration stages. The red arrows shows the way of the <sup>3</sup> He from its way from the top to the <sup>3</sup> He bath and the way out out again. On the right hand side, the design shows the outflow of <sup>3</sup> He when it is evaporated during the cooling process of the UCN guide. [12]	25
24	a) depicts the schematic built of the very first counterflow HEX in the cryostat. b) shows the construction of the second and third HEX with the implement of a thermal shield. In this counterflow HEX are two <sup>4</sup> He upstreams cooling the one <sup>3</sup> He downstream	_0
25	and the thermal shield	25
26	both plans.[12]	26
20	baths. The different colored arrows point the direction and medium of the flow. [12] .	27
27	The figure shows the simplified second cooling stage with the counter flow HEX between the two ${}^{4}\text{He}$ pots. [12]	27
28	Both parts illustrates the cryostat with numbered points after each refrigeration step. The points are numbered from beginning to end in ascending order and each one represent different properties of the Helium in the pipe. Section A) refers to the <sup>3</sup> He system and section b) to the <sup>4</sup> He system. The latter system has two outlets and therefore, the <sup>4</sup> He system is separated in the warmer outlet, path "A" and the colder outlet, path "B"	20
29	The schematic built of HEX4, HEX5 and HEX 6, with the relating heat flows. The origin of flow "A" and "B" are illustrated in Fig. 28. The thermal shields of HEX4 and	25
30	HEX5 are neglected	32
31	rising HEX4 exit temperature. The figure shows schematically the outlet of the <sup>3</sup> He. It gets vaporized in the <sup>3</sup> He pot and on the way to the pump the vapor passes section $1 - 5$ where section 2 and 4 are $90^{\circ}$ bendings. Along section 1 are three temperature clamps fixed that heat the up	38
32 33	The schematic laminar and the turbulent regime. [9]	41 42 48
	and part of should be volume non at the pump via an merodoing tupor probation .	10

34	The pipe design for a outlet that is separated in two sections with two different diameters. D1 describes the diameter of the first section of the outlet and D2 describes the diameter			
	of the remaining pipe segments	51		
35	The illustration depicts the schematic helium cryostat. The yellow rectangle indicates			
	HEX1 where the issue of Kapitza conductance between liquid helium and the copper			
	pipe at two interfaces occurs. $[12]$	53		
36	This Graph shows a example for the behavior of the temperature of He-II (blue line)			
	and copper (red line) at an Cu-He-II interface. The difference between the temperatures			
	over the time is caused due to the Kapitza conductance. [17]	54		
37	The figure depicts above the schematic cross section of HEX1 with one IP-He-Cu and			
	one Cu-3He interface. The heat flux is depicted from IP helium towards the <sup>3</sup> He. The			
	lower diagram shows the behavior of the temperature through the upper cut including			
20	the temperature changes caused by the Kapitza resistance. $[17]$	$\overline{33}$		
30	interface with different surface conditions. The red filed area shows the region where the			
	measurements of Kapitza conductance at different surface conditions were. The green			
	line shows measurements of Kapitza conductance for a very well cleaned surface and			
	the purple line measurements of Kapitza conductance for a very bad surface condition.			
	The vellow line refers to experimental data, that simulated the HEX in order to get			
	a estimation for Kapitza conductance that is possible at TRIUMF [17]. The blue line			
	shows the theoretical upper limit of the Kapitza conductance, called phonon radiation			
	limit, the red line shows the theoretical lower limit, the Khalatnikov theory. [16, 17] .	58		
39	The temperature measurements in a ${}^{3}$ He bath and a ${}^{4}$ He bath over a variating heat			
	input are displayed. The non-filled circles are equal to the measurement points M3 and			
	the filled ones regard to the measurement point M4. [19]	59		
40	The design of the old Copper heat exchanger with fins on both sides, it was used in the			
	vertical UCN source. [20]	60		
41	The old design of HEX1 is displayed schematically. The upper point M3 is the tempera-			
	ture measurement point in the "He bath and the lower point M4 is the measurement			
	point in UCN guide. The orange part between the two henum section points the copper	60		
49	The chart shows the result of temperature measurements from last Nevember at TRIUME	00		
42	The praph depicts the behavior of the temperature difference between two measurement.			
	points one inside of the UCN guide and one in the <sup>3</sup> He section over different an			
	increasing amount of heat input. The "base" values are regarding to the activation of			
	the heater. The "sat" values mean after saturation of the temperatures with the heater			
	on. [18]	61		
43	Part a) depicts schematically the heat transfer with natural convection at a surface			
	where heat is applied from underneath to the surface. In this case, warmer liquid He-I is			
	flowing up and mixing with the colder section above. Next to it, section b) shows the			
	nucleate boiling regime at the same experimental procedure with a larger amount of			
	heat supply. Here are forming bubbles at the heated surface which start rising to the			
	top. Item c) is film boiling which occur under even higher energy input. Because of the			
	large energy input, bubbles are forming and detaching very fast. So that they form the			
	pictured vapor film between surface and He-I. [6]	62		

44	[25] The diagrams show the temperature difference between Copper and <sup>3</sup> He during	
	neat input to the Copper side. The experiment was under saturated vapor pressure.	
	Part a) is the temperature difference including the Kapitza conductance and part b) is	
	the temperature difference without the proportion of the temperature change caused	
	by Kapitza conductance. The blue dots refer to the regime of natural heat convection,	
	the red points are according the nucleate boiling regime and the green dots at the top	
	belong to a third regime, the film boiling. The empty dots refer to the same experiment	
	with an decreasing heat input, but this observation is neglected in this context. It is	
	noticeable that a switch in regime a significant change in temperature causes. The	
	arrows between different point of separate regimes show that the temperature change	
	for the same applied heat isn't the same for different regimes	64
45	The figure shows the schematic cross section at HEX1 without fins. On the left hand	
	side are the materials of the pipe, Copper and Nickel, and the mediums of the of the	
	different parts. The heat flux is pointed by the red arrow. The green point (M4) is a	
	temperature sensor in the UCN guide and the blue point (M3) is a temperature sensor	
	in the <sup>3</sup> He pumping circle. [18] $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$	66
46	Part a) illustration of the cross section of the cylindrical HEX. The blue circle refers to	
	the inner radius and inner surface of the HEX. The outer green circle depicts the outer	
	radius and surface. The fins around the duct are indicated by the green circle. The view	
	in section b) shows the HEX from the side with the fins that are like rings around the	
	m section of shows the HER from the side with the find that are like fings around the	66
	mam tube	00

# List of Tables

$\frac{1}{2}$	Classification of neutrons in terms of their free kinetic energy. $\dots$ $\dots$ $\dots$ $\dots$ The heat properties of <sup>3</sup> He at every point of the cryostat are shown. The points refer	3
	to section a) of Fig. 28 and the state indicates the phase of the <sup>3</sup> He where "V" means	30
3	The heat properties of <sup>4</sup> He in terms of outlet " $\Lambda$ " are listed. The property points refer	30
9	to section b) of Fig. 28 and the state indicates the phase of the <sup>4</sup> He where "V" means	20
4	vaporous and "L" means liquid. $\ldots$	30
4	refer to section b) of Fig. 28 and the state indicates the phase of the <sup>4</sup> He where "V" means vaporous and "L" means liquid	31
5	The list refers to the cooling power of the two <sup>4</sup> He baths in order to obtain the assumed	01
	$^{3}$ He temperature after the pot.	31
6	Heat in- and output of the counterflow HEX4, HEX5 and HEX6. The inflowing <sup>3</sup> He is refrigerated by the two different outlets of <sup>4</sup> He. The last column indicates the total energy balance at the certain HEX. A positive value indicate heat input and a negative	
	value heat output.	32
7	Heat in- and output of the counterflow HEX4, HEX5 and HEX6 for only one counter flowing path, outlet B of Fig. 29. To obtain the required cooling, either the temperature at point "B3" is increased or the temperature of point "B4" is decreased, compare points to Fig 28 b). $Q_{3\text{He}}$ indicates the necessary cooling of the <sup>3</sup> He and the last column indicates the total energy balance at the certain HEX. A positive value indicate heat	
	input and a negative value heat output.	33
8	Heat in- and output of counterflow HEX3. Here are the inflow of ${}^{3}$ He and ${}^{4}$ He are cooled	
	by vaporized <sup>4</sup> He of the 1K pot. $\ldots$	33
9	Heat in- and output of counterflow HEX2. The ${}^{3}$ He that flows in gets cooled by the	
	vapor that gets pumped away.	34
10	This is the reference table for the following assumptions. The mass flow and its respectively volume flow for an unchanged cryostat are listed. The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate $\left(\frac{m^3}{h}\right)$ , and the non-pumping systems, where the volume flow is stated in the liquid	
	consumption $\left(\frac{L}{h}\right)$ .	34
11	The mass flow and its respectively volume flow of all affected flow systems are shown for a removal of HEX2. The proportion by which the flows increased compared to the design with HEX2 (Table 10) is listed. The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate $(\frac{m^3}{2})$ , and the non-pumping	
	systems, where the volume flow is stated in the liquid consumption $(\frac{L}{L})$ .	35
12	The mass flow and its respectively volume flow of the ${}^{4}$ He system are shown for a removal	
	of HEX3. The proportion indicates by which amount the flows increase compared to the	
	design with HEX3 (Table 10). The volume flow differs between the pumping systems,	
	where the volume flow is specified in the pumping rate $(\frac{m^3}{h})$ , and the non-pumping	
	systems, where the volume flow is stated in the liquid consumption $(\frac{L}{h})$ .	36
13	The mass flow and its respectively volume flow of all affected flow systems are shown for a removal of HEX2 and HEX3. The proportion by which the flows increased compared to the design with both HEX ((Table 10)) is listed. The volume flow differs between the numerical systems, where the volume flow is specified in the numerical system.	
	non-pumping systems, where the volume flow is specified in the pumping rate $(\frac{m}{h})$ , and the non-pumping systems, where the volume flow is stated in the liquid consumption $(\frac{L}{h})$ .	37

14	The mass flow and its respectively volume flow of the <sup>4</sup> He system are shown for a removal of HEX4. The proportion indicates by which amount the flows increase, compared to the design including HEX4 (Table 10). The volume flow differs between the pumping systems,	
	where the volume flow is specified in the pumping rate $\left(\frac{m^{\circ}}{h}\right)$ , and the non-pumping	
	systems, where the volume flow is stated in the liquid consumption $(\frac{L}{h})$	37
15	The heat input into the 4K bath and its affected volumetric flow are listed in terms of the <sup>3</sup> He temperature after leaving HEX4. The inlet consumption is stated for the	
	construction with and without HEX2.	38
16	The mass flow and its respectively volume flow of all affected flow systems are shown for	
	a removal of HEX2, HEX3 and HEX4. Furthermore, the proportion by which the flows	
	increased compared to the design with both HEX (Table 10) is listed. The volume flow	
	differs between the pumping systems, where the volume flow is specified in the pumping	
	rate $(\frac{m^3}{h})$ , and the non-pumping systems, where the volume flow is stated in the liquid	
	consumption $(\frac{L}{h})$ .	39
17	The volume flow of the individual flow systems for the general built with all HEX and the	
	built with the removal of one or more HEX. The flow is distinguished between pumping	
	systems, where the volume flow is specified in the pumping rate $\left(\frac{m^3}{h}\right)$ stated, and the	
	non-pumping systems, where the volume flow is stated in the liquid consumption $(\frac{L}{h})$ .	40
18	The wall temperature increase at the different parts of the <sup>3</sup> He outlet. The bended part,	
	2 and 4, are listed with their effective length. The sections are depicted in Fig. 31	44
19	[13] The ratio between radius and diameter and its respective resistance coefficient.	45
20	Pressure drop over an rising <sup>3</sup> He flow in the <sup>3</sup> He outlet with a diameter of 0.1 m	46
21	Pressure drop in the <sup>3</sup> He outlet with a constant helium flow of 0.5 $\frac{3}{s}$ and an increasing	10
22	diameter of the outlet channel.	48
22	Pressure drop for a pressure of 377.8 Pa at the beginning is examined in the five different	
	sections of the "He outlet (depicted in Fig. 31), for two different helium now rates and	40
0.0	The total manuary dameter of the duct.	49
23	The total pressure drop in the "He outlet for two different fieldum now rates and an	
	277.8 Do at the baginning of the outlet and is shown with the approximated warm density	
	and the required pumping volume	50
24	The pressure drop in a outlet that is designed with two parts of different diameters, as	50
24	seen in Fig. 34. The reference pressure at the beginning is 377.8 Pa, with this value is	
	the pressure drop for different duct diameters with its effected vapor density and volume	
	flows at the nump stated	50
25	The pressure drop for different outlet designs under different helium flow rates is listed	00
20	Which diameter influences which outflow section is shown in the Fig. 34. The reference	
	pressure at the beginning is 377.8 Pa	52
	r	

# 1 Introduction

#### 1.1 Motivation

Nowadays one of the most interesting questions is about how the whole Universe came into being. To get closer to this explanation, scientists try to analyze every small detail they know about the space. Therefore many inexplicable issues occur that cant be explained with todays findings. One of these topics is the huge asymmetry of mater (baryon) and anti mater (antibaryons) in the entire Universe [1]. Observations showed that the amount of matter is very huge compared to the total number of anti mater. Consequently to get the answer to the origin of the Universe the question about this imbalance has to be solved.

Reference [1] explains, that the scientist Andre Sakharov discovered in 1967 that the Baryon asymmetry is strongly affected by the violation of the charge-parity (CP) symmetry. However, the known CP-violation of the Standard Model (SM) is too small to explain the huge disparity of baryons and antibaryons in the Universe. Because of this discovery it was realized that there must be another source of CP-violation, which justifies the whole Baryon asymmetry. To notice a new origin of CP-violation it is need to look at a process where the influence of the SM is very small, but other influences which are not included in the SM are big enough to be measured. A good example for this case is the electric dipole moment of the neutron (nEDM). A neutron is usually neural and has therefore a nEDM equal zero. However, if the inner positive and negative charge distributions are a little moved, the nEDM gets unequal zero. As soon as a particle has an EDM unequal zero the CP-symmetry is violated. Because under time (T) reflection, the charge distributions stay constant but the spin turns backwards. So the direction of the EDM will change, what violates T-symmetry. According to CPT-symmetry, a violation of T-symmetry has always the consequence that CP-symmetry is violated too [1].

Summarized it is needed to measure the nEDM to evidence the CP-violation of neutrons. This insight will indicate a new source of CP-violation and gets the science closer to the question, how the Universe came into being.

### 1.2 Neutron properties and production

Now that we understand the motivations for UCN production, let us have a closer look at neutrons and the production process itself.

#### Nucleon structure

Neutrons together with protons are part of every atomic nucleus. The mass of a single neutron is  $1.67 * 10^{-27}$  kg what equals approximately the mass of a single proton [26]. Electrons complement the structure of the atoms, the mass of electrons is about 1800 times smaller stated than that of the particles in the nucleus. The three particles do not only differ in mass but also in charge. Electrons have the negative elementary charge (e), while nucleons are made out of up (u) and down quarks (d) with different electric charges [2]. Protons are built from two up and one down quark and neutrons consist of one up and two down quarks (Fig 1). Up quarks have an electric charge of +2/3 e and down quarks a electric charge of -1/3 e. Consequently, the total charge of a proton is +1 e and the charge of a neutron is zero.



Figure 1: [2] Composition of a proton and a neutron.



Figure 2: [3] An example of the charge displacement measurement. If the size of a neutron were equal to the size of the earth, then the measured upper limit for the charge displacement would be 1  $\mu$ m.

#### nEDM

For a slight displacement of the internal charge distributions, the neutron gets a non-zero EDM. By considering nucleon structure we can describe this phenomenon more clearly. The charges are the one up and the two down quarks in the neutron, so if the combined negative charge center of the down quarks and the positive charge center of the up quark are slightly moved, the EDM of the neutron would be unequal zero. An upper limit of the EDM of  $1.2 \times 10^{-25}$  cm has been measured, but the SM predicts a much smaller EDM of about  $10^{-31}$  cm [5]. Therefore, the current goal is to make a more accurate measurement to verify the prediction of the SM. Figure 2 illustrates the difficulty of this issue: if a neutron were the size of the earth, the current measurements would be able to measure the displacement of the charges to 1  $\mu$ m. However, a more precise measurement must be made to be able to evaluate the predicted charge displacement of the SM.

#### Neutron classification

Neutrons can be classified into different types according to their temperature. The temperature of a neutron is defined by its free kinetic energy [39]. Table 1 shows the different

<b>Energy</b> $[meV]$	$\mathbf{v} \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$	Wavelength $[nm]$	Energy classification
$< 3*10^{-4}$	< 7.6	> 52	ultra cold
$3*10^{-}4 - 0.05$	7.6 - 98	4 - 52	very cold
0.05 - 10	$98 - 1.4 * 10^3$	0.3 - 4	cold
10 - 100	$1.4 * 10^3$ - $4.4 * 10^3$	0.09 - 0.3	thermal
100 - 500	$4.4 * 10^3$ - $9.8 * 10^3$	0.04 - 0.09	$\operatorname{hot}$
$500 - 10^5$	$9.8 * 10^3$ - 138 $* 10^3$	$3*10^{-}3 - 0.04$	epithermal

Table 1: Classification of neutrons in terms of their free kinetic energy.

energy classifications of neutrons, neutrons are ultra cold when their kinetic energy is below  $3 * 10^{-4}$  meV and respectively their velocity is smaller than 7.6  $\frac{\text{m}}{\text{s}}$ .

Moreover, neutrons are affected by all four fundamental forces [4]: the gravitational, electromagnetic , weak and strong interactions. The weak interaction causes a UCN to decay into a proton, an electron and an electron antineutrino after a lifetime of  $880.3 \pm 1.1$  s.

$$n \to p + e^- + \bar{v}_{\rm e} \tag{1}$$

In addition, the strong interaction makes it possible to store UCNs in a vessel. Because of the low energy of the neutrons, the vessel material can be chosen so that the Fermi energy of the wall is higher than the energy of the neutrons. Consequently, the neutrons are not able to overcome the Fermi energy and remain trapped inside of the vessel.

#### 1.3 UCN production

UCN production is the main goal of this project, so we will go through the different steps of the process in detail.

#### Beam line and target

We have seen, that free neutrons decay with a lifetime of roughly 880 s. So to store neutrons as long as possible, we need to liberate neutrons from nuclei. As described by [35], a proton beam is focused onto a neutron rich spallation target - Triumf uses tungsten - so that a few nucleons are produced. Additionally, the whole nuclei is excited and as can be seen in Fig. 3, the nuclei get rid of this energy by emitting several energetic particles, like neutrons, protons and alpha particles. The particles that are released by the nucleus are very energetic and collide with other surrounding nuclei. In this way, a chain reaction occurs, so that more and more nuclei emit particles. Then, the neutrons must be filtered out and cooled down.

#### Moderator system

The moderator system at Triumf [12], is intended to filter and slow down the neutrons while minimizing the number of neutron captures. The cooling is done through the collision of neutrons with moderator



Figure 3: [36] The process of a proton (p) colliding with a tungsten nuclei, producing neutrons (n) and protons.

atoms, so that the neutrons give a proportion of their energy to the moderator atoms, which then transfer the energy to their neighboring atoms. Furthermore, the absorption of neutrons is prevented by using moderator materials with a small neutron-absorption cross section, like heavy water (D<sub>2</sub>O) and liquid deuterium (LD<sub>2</sub>). The moderator system at Triumf, depicted in Fig. 4, is built of several different materials. The outer moderator is graphite and lead at room temperature and serves to filter the unwanted particles, like protons, out. Thereafter comes a section of heavy water also at room temperature, and next, a section of liquid deuterium at 20 K. At last, the neutrons enter the 1 K cold superfluid helium system, where they are downscattered to the ultra cold state under emitting an phonon, which carries away most heat. The process is displayed in Fig. 5. To prevent the two cold moderators, liquid deuterium and superfluid helium, from getting warm from the heat load of the neutrons, they must constantly cooled. In the case of deuterium, the liquid is constantly exchanged, so that cold deuterium is always available. On the other hand, the superfluid isn't replaced. Here the heat is transfered to a helium cryostat, which transfers the heat out.

After the neutrons are ultra-cold, the energy of the neutrons is lower than the Fermi energy of the nickel surface of the wall, therefore the neutrons will experience a total reflection at the guide wall under every angle of incidence. The UCN move under this reflection through the UCN guide, pictured in Fig. 8, to the experimental area.

In summary, as shown in Fig. 6, the neutrons are produced from tungsten nuclei by a proton beam and then they are filtered and downscattered by several moderators. When they have reached the last moderator (the He-II bath) the neutrons are ultra-cold so that they are unable to go through the material of the wall, so they will be reflected at the walls until they reach the experimental apparatus.

#### 1.4 UCN loss

Finally, let us consider the UCN loss in the production process. In [38], the loss is defined as

$$\frac{1}{\tau_{\text{total}}} = \frac{1}{\tau_{\text{He-II}}} + \frac{1}{\tau_{\text{vapor}}} + \frac{1}{\tau_{\text{abs}}} + \frac{1}{\tau_{\text{wall}}} + \frac{1}{\tau_{\text{leak}}} + \frac{1}{\tau_{\text{decay}}}$$
(2)

where  $\tau$  is the lifetime of the neutrons and the index describes the different sources of UCN loss. These various factors are explained in the following sections.

#### Up-scattering in the superfluid

As mentioned before, cold neutrons are down-scattered by the superfluid <sup>4</sup>He to become ultra-cold neutrons. In this process the majority of the neutrons kinetic energy is emitted as phonons. It might



Figure 4: [12] The proton beam hits the target; consequently the target starts emitting particles which enter the moderator system. The moderator system includes a layer of graphite and lead, heavy water, liquid deuterium and d superfluid helium.



Figure 5: [36] The downscattering process of a neutron, from the cold to the ultra cold stage.



Figure 6: [37] The UCN production process, from the emitted high energy neutrons through the different moderator systems and the final cooling in superfluid helium. The scale on the right hand side shows the temperature and velocity of the neutrons through the different steps.



Figure 7: [38] The neutron energy loss rate from the phonons over an increasing temperature.

happen that one of these phonons hits another neutron and transmits some energy to it. After the neutron gains energy it is brought back to the cold state and is allowed to leave the vessel because it can overcome the Fermi energy of the wall. The neutron lifetime from this effect is assumed to be

$$\frac{1}{\tau_{\rm He-II}} = 0.008 \, T^7. \tag{3}$$

This function is depicted for the approximate temperature range, in Fig. 7 and we see that the loss rate is strongly dependent on the temperature of the superfluid helium.

### Inelastic scattering by helium vapor

This kind of UCN loss is, like before, caused by an energy gain of the neutrons, but here the energy increase is caused by helium vapor. As can be seen in Fig. 8, the second part of the UCN guide is filled with helium vapor and when the UCN pass this section it may be that it gets inelastically scattered



Figure 8: [12] The UCN guide, where the neutrons enter on the right side in the liquid Helium, pass the heat exchanger, and finally leave through a guide filled with helium vapor.

and gains energy from the vapor atoms. Just like before, this energy allows the neutrons to go through the walls. In total, the UCN loss by inelastic scattering is written as

$$\frac{1}{\tau_{\rm vapor}} = n\rho_{\rm s}v\tag{4}$$

where n is the number density of <sup>4</sup>He vapor,  $\rho_s$  is the coherent scattering cross section for helium and v is the velocity of a Helium gas molecule.

#### Wall loss

Wall loss is another source of UCN loss which can be partly described by energy gain of the neutrons. Similar to the helium atoms, wall atoms can also scatter the UCN inelastically leading to a loss of neutrons. In addition, neutrons are absorbed by the nuclei of the wall surface. The two different kinds of wall loss can be described in one formula:

$$\frac{1}{\tau_{\text{wall}}} = \frac{\bar{\mu}\bar{v}S}{4V}.$$
(5)

In this equation,  $\bar{\mu}$  is the effective loss rate per bounce,  $\bar{v}$  is the averaged UCN velocity, S is the surface area of the container and V is the volume of the vessel.

#### Neutron absorption

Furthermore, the UCN guide is filled with <sup>4</sup>He because <sup>3</sup>He has a large neutron absorption section. For this reason, <sup>4</sup>He instead of <sup>3</sup>He is used in the UCN guide, because it has an absorption rate of zero. However, in reality <sup>4</sup>He can't be produced without a small fraction of <sup>3</sup>He, so there is always a small number of UCN absorbed by these <sup>3</sup>He atoms. The absorption loss rate is defined as

$$\frac{1}{\tau_{\rm abs}} = n_{\rm 3He} \rho_{\rm a} v \tag{6}$$

where  $n_{rm3He}$  is the number density of <sup>3</sup>He,  $\rho_a$  is the neutron absorption cross section and v is the velocity of the UCN. Since the neutron absorption cross-section is a function of  $\frac{1}{v}$ , the total absorption rate is independent of the UCN velocity.

#### Leakage through gaps

Another loss comes from UCN leakage through holes and gaps. Neutrons are very small particles, so they are able to leave through very tiny gaps or holes. This kind of loss is given by the equation of wall loss for a effective loss rate per bounce of 1. The loss rate is written as

$$\frac{1}{\Gamma_{\text{leak}}} = \frac{vS}{4V} \tag{7}$$

where S is the area of the gap.

#### Neutron $\beta$ -decay

At last, we noticed that due to the weak interaction, neutrons decay after a certain time. When this happens, neutrons are also lost and can't be used for their experimental purpose. The neutron lifetime is about 880 s, so the loss rate is

$$\frac{1}{\tau_{\rm decay}} = 1.14 * 10^{-3} \,\mathrm{s}^{-1}.\tag{8}$$

## 2 Superfluid helium

#### 2.1 Helium phases

Reference [6] describes that superfluid is separate phase just like the solid, fluid and vapor phases. The phase diagram of <sup>4</sup>He in Fig. 9 shows that the superfluid Helium (He-II) and liquid Helium (He-I) are different phases and that the He-II phase is the energy state with the lowest possible energy. The line that splits these two liquid phases is called the  $\lambda$ -line. For instance, at a pressure of 100 kPa, the transition from vapor to liquid is at a temperature of 4.2 K and the transition from liquid to superfluid helium is at a temperature of 2.17 K. Moreover, we can see that helium has no solid phase under ambient pressure. For solidification, a pressure of more than 2.5 MPa would be required, due to the large zero point energy of helium. The high pressure required for solidification is the reason why helium has no triple point, where the three phases coexist. One reason for the large zero point energy, mentioned by [30], is that helium has the lowest boiling temperature and critical point of all liquids.

#### 2.2 Properties of superfluid helium

Now, we want to take a closer look at the superfluid phase itself, so, we will analyze the experiments of Alfred Leitner regarding superfluid helium [34].

#### Viscosity experiments

When a normal liquid flows through a pipe, the walls of the tube will resist the flow, due to the interatomic van-der-Waals forces between the moving atoms of the fluid and the resting atoms of the wall. For a fluid with a strong force of attraction, the flow will be very slow. In contrast, for substance



Figure 9: [6] Phase diagram of <sup>4</sup>He where the  $\lambda$ -line describes the transition between He-I and He-II.

with a low force of attraction, the movement will be faster. Consequently, a high viscosity fluid will have a slow flux and a low viscosity fluid will have a fast flux. We will look at some tests of the viscosity of He-I and He-II, to determine whether the force between the helium atoms in He-II is strong or not.

First viscosity experiment For this experiment, a beaker with a porous bottom layer is filled with He-I. The viscosity of He-I is very low, consequently the capillaries in the bottom disc must be fine enough to prevent the fluid from passing through under its own weight. In Fig. 10 it can be seen, that the boiling He-I is viscous because it stays in the beaker and doesn't leak out. The helium is at its boiling temperature of 4.2 K, therefore bubbles that are constantly rising to the top. Then, the helium is cooled down to the  $\lambda$ -point, where the phase changes from He-I to He-II. As soon as the transition temperature is reached, the bubbling stops abruptly and the liquid pours out through the tiny holes in the bottom layer. The cessation of boiling means that the phase has changed and the helium is now superfluid. Moreover, the sudden leak rate increases for a decreasing temperature and is independent of the size of the capillaries. This type of flow is called superflow.

In conclusion, at first we noticed that a slow flow implies a large viscosity, so He-I must be viscous because the interatomic forces have not allowed any movement from the helium through the porous bottom disc. On the other hand, He-II is able to leak out, so the interatomic forces must be very small or zero. Consequently, He-II must have a viscosity that is very small or zero. Additional experimental attempts to find the viscosity of He-II were inconclusive, since the uncertainty of the measuring techniques were too large for such a small viscosity. It is believed that the viscosity must be zero.

**Second viscosity experiment** Another important experiment involves a copper cylinder and a wooden paddle wheel mounted in a liquid Helium bath. As Fig 11 a) suggests, the paddle wheel is fixed above the cylinder, they are not connected and both are free to rotate on a vertical axis. At first, the cylinder is set in motion by the four coils of the induction motor, depicted in Fig 11 b), from the



Figure 10: [34] The behavior of He-I and He-II in a beaker with a porous bottom.



Figure 11: [34] The construction of the second second viscosity experiment, in a) are the copper cylinder and the wooden paddle wheel seen from the side and b) is the top view of the whole experiment, including the four coils of the induction motor.



Figure 12: [6] The experimental construction of the fountain effect. The liquid in the sphere gets heated, consequently the liquid squirts out through the tube.

outside of the bath. Both He-I and He-II are non-conducting, so they aren't affected by the induction from the motor. The wheel isn't affected by the motor either, but it can turn around the vertical axis as well. So if the liquid helium is viscous or in other words, if interatomic forces act between the liquid atoms, the paddle wheel will also start to move through the flow of the liquid.

At first, the cylinder is run in the warm He-I phase. After a few seconds the wooden paddle wheel starts rotating as well. Consequently, the helium atoms must get dragged by the rotating atoms of the cylinder surface, which attracts the adjacent helium atoms, until the paddle wheel is set in motion. Hence, we can say that He-I is viscous.

Afterwards, the helium in the bath is cooled down and enters the colder He-II phase. Now, the same procedure is repeated and as before, the wooden paddle wheel starts to rotate. Accordingly, interatomic forces must also act on He-II, and therefore superfluid helium must be viscous.

The observations in viscosity experiment I and II dont match each other. For He-I the results are consistent, so it can be concluded that the He-I phase is viscous. In contrast, the observations of He-II diverge. The test with the capillary flow says that He-II has zero viscosity, as opposed to the rotating cylinder method, that concludes that He-II is viscous. All known classical liquids are known to behave the same way in capillary flow as in bulk flow. Consequently this paradox comportment must be explained in a new and unique way. So, superfluid helium must be thought as a dual and not single liquid. This dual liquid must be imagined with two independent components, this is called the two-fluid model of superfluid helium. One of them is called the normal component, it is the part that is viscous. The effect of this component is that, in the second viscosity experiment, the liquid helium is dragged by the cylinder and consequently the wooden paddle wheel is accelerated. But, due to its viscosity, the normal component cant flow through the porous bottom layer in Experiment I. The second part that shows zero viscosity is called the superfluid component. This part flows through the narrow channels of the porous bottom layer in experiment I because of its lack of viscosity. On the other hand, the superfluid component does not participate in experiment II with the rotating cylinder. This model is able to describe the shown behavior of superfluid Helium. Whether it is correct requires further experimental tests.

#### Fountain effect

Another unique behavior of superfluid Helium is called the fountain effect, described by both Alfred Leitner and van Sciver [6]. The tube of Fig. 12 is designed so that after a short segment, the tube



Figure 13: [6] All beakers and baths are filled with He-II. Each part shows the movement of the rolling film for different submerged depths and the various fillings of the beaker.

opens into a sphere and then merges into a capillary. The pipe section underneath the sphere is filled with a very fine powder, which acts as very thin capillary channels. In order to keep this powder in place, cotton tufts are placed above and below the powder.

This device is submerged in liquid helium, with a temperature below the  $\lambda$ -point. Then, the object fills with liquid until the liquid levels are in equilibrium. This happens because of the earlier described superflow of superfluid helium. We notice also that this must be the superfluid component which is flowing through the thin channels. Later on, the heater is turned on, which heats the liquid inside of the bulb. We observe that more liquid helium flows into the tube and the liquid level in the tube rises until the liquid spurts out of the top. The reason for the rising helium will be analyzed later. At first, we will focus on the additional helium flowing into the tube. This behavior can be described as cold liquid Helium flowing to a place with a higher temperature. This phenomenon violates the second law of thermodynamics, where heat should always flow from a place of higher to a place of lower temperature. This must mean that the superfluid component of He-II carries no heat and no thermal energy. Consequently, all heat energy in He-II has to be contained in the normal component of He-II. Then the helium is not heated by the heater but it is transformed from the superfluid component, by the heat, to the normal part.

Now let's return to the observation of the rising helium in the capillary while the heater is on. This observation can be understood using London's equation, which is defined as

$$\Delta p = \rho s \Delta T \tag{9}$$

where  $\rho$  is the density, s the specific entropy and  $\Delta T$  the temperature increase. In this experiment with He-II, the temperature is rising what causes the pressure to rise according to Eq. (9). Consequently, the liquid level rises until the liquid squirts out.

#### Rollin film

The same anomaly can be observed using a similar experimental construction of [6], seen in Fig. 13. There are beakers filled with different levels of He-II and submerged into a bath of He-II. In the first case Fig 13 a), the beaker is submerged into the bath so that the liquid level of the bath exceeds the liquid level inside of the beaker. Then, the liquid level in the beaker will rise and the liquid level of the bath will decrease until the liquid levels inside and outside are equal. In other words, the liquid starts to flow over the wall into the beaker with the lower level and ends when the levels are equal. This film is called the rolling film with a thickness of only a small friction of one micron. Secondly, in Fig. 13 b), the same process is repeated except that the beaker is submerged so that the liquid levels are the same. For this configuration no liquid flow can be seen. At last, in Fig. 13 c), the beaker is submerged so that the liquid level inside exceeds the fluid level outside. As in the first trial, the higher liquid level inside decreases and the lower fluid level outside increases until the levels are the same. In another experiment [34], there, the beaker isn't submerged but is held above the liquid bath. As before the liquid levels start to equalize. Since an equilibrium can't be reached, all of the liquid flows out of the beaker.

In summary, as in the previous experiment, the liquid flows upwards until the levels in both vessels are equal. Similarly to the fountain effect, this observation be explained by pressure of the helium. As soon as one liquid level rises above the other one, the hydrostatic head or in other words, the pressure of the higher liquid forces the liquid to flow until the levels are equal. In other words the pressure of the liquid in the beaker with the higher liquid level exceeds the pressure in the other one. Consequently, the higher pressure causes the liquid to flow into the vessel with lower pressure. The hydrostatic head between the vessels is defined as

$$\Delta p = \rho g \Delta h \tag{10}$$

where  $\rho$  is the density of He-II, g is the gravitational constant and  $\Delta h$  is the difference in the liquid levels. Since the  $\Delta h$  is zero for equal fluid levels, the flow stops after reaching equilibrium. In total, according Eq. (10), the different liquid levels cause a pressure difference in the liquids so that the helium flows up the wall until the liquid levels are equal.

#### Second sound

Lastly, let us consider another noteworthy property of superfluid helium related to heat transfer. Before presenting details, we will first look at heat transfer in common systems. Usually heat spreads in solids, liquids or gases through a diffusion process of the warmer and colder particles.

To examine the heat transfer in He-II experimentally, heat is applied from a resistor to a bath of liquid helium and at the far end, the transfered heat is recorded by a second resistor, as illustrated in Fig. 14. The disc-shaped resistors are aligned with each other and both have a carbon layer at the surface that guarantees them a good thermal contact with the fluid. The liquid is heated by the transmitting disc by sending electric current through it. Then, the heat moves to the receiver, where it causes a temperature rise in the resistor. Consequently, the electric resistance of the receiver changes and a small direct-current, which flows through it, will change. In addition, the two discs are connected to a dual trace oscilloscope which will display the heat impulse at the transmitter and the change in the resistance of the the receiver. For diffuse heat transfer, the resistance of the receiver is expected to increase slowly over the time. However, the oscilloscope shows that about 3 ms after the pulse has left the transmitter, a resistance pulse at the receiver is recorded. This fast transmission and the



Figure 14: [34] Schematic built of the second sound experiment.

recorded pulse are evidence, that the heat transfer must be a wave process. This phenomenon is called the second sound of superfluid helium and the propagation speed is called the speed of second sound. Moreover, the superfluid phase is the only known phase with a characteristic like this.

The heat transfer in superfluid helium, in form of a wave, differs completely from conventional systems. This so-called second sound, spreads the heat in He-II very quickly, making He-II the best known heat conductor.

### 3 Cryostat design

The helium cryostat is a fundamental part of the UCN production process, it ensures that the IP helium in the UCN guide is at the required temperature and consequently the UCN loss is kept small. So let us have a closer look at the helium cryostat.

#### 3.1 Comparison of helium isotopes

Before we start, let's clarify, which coolant is best. As discussed in Pobell [14], for very low temperatures, helium is the only liquid that does not change into the solid phase under its own vapor pressure, so it is the most suitable refrigerant for the cryostat. Helium has two stable isotopes, <sup>3</sup>He and <sup>4</sup>He, that can be used as coolant. In terms of the overall goal, to transfer as many heat out of the system as possible, we want to compare <sup>3</sup>He and <sup>4</sup>He in respect to their heat flux. In this sense, the heat flux can be described via

$$\dot{Q} \propto L P_{\rm vap}$$
 (11)

where L is the heat of evaporation and  $P_{\text{vap}}$  is the vapor pressure. Therefore, let us compare the heat of evaporation and the vapor pressure for <sup>3</sup>He and <sup>4</sup>He

Regarding the heat of evaporation we can compare the plots of the isotopes in Fig. 15 a). Here we



Figure 15: Section a) depicts the heat of evaporation of <sup>3</sup>He and <sup>4</sup>He over the temperature. Part b) illustrates the vapor pressure of <sup>3</sup>He and <sup>4</sup>He over the temperature as well.[14]

see, the heat of evaporation of <sup>3</sup>He is over the whole temperature range much lower than the heat of evaporation of <sup>4</sup>He. For example, at a temperature of 1 K the latent heat of <sup>4</sup>He is about two times bigger than that of <sup>3</sup>He. With respect to Eq. (11) we can say that in terms of the heat of evaporation, the heat flux of <sup>4</sup>He is greater than for <sup>3</sup>He.

Moreover, Fig. 15 b) illustrates the vapor pressure of both isotopes for temperature up to 5K. In this figure, the vapor pressure of <sup>3</sup>He is much higher than that of <sup>4</sup>He. For a temperature of 1 K, <sup>3</sup>He has a vapor pressure that is about 100 times bigger than that of <sup>4</sup>He. Applied to Eq. (11), it is correct to say, that regarding the vapor pressure, <sup>3</sup>He has the higher heat flux.

In summation, due to the greater vapor pressure of  ${}^{3}$ He, compared to the small difference in the latent heat, the cooling power of  ${}^{3}$ He is much bigger than that of  ${}^{4}$ He.

In addition, an issue about using <sup>3</sup>He is, that the production process of it is much more complicated than for <sup>4</sup>He. According van Sciver [6], the cost for one liter <sup>3</sup>He at standard pressure and temperature are between 100 \$ to 1000 \$ per liter. On the other hand, [28] stated the costs for regular <sup>4</sup>He of only 35 \$ per liter. However, due to the lower heat flux for the use of <sup>4</sup>He, the use of it will be much higher than for <sup>3</sup>He, so that the total costs for <sup>4</sup>He will be high as well.

All in all, <sup>3</sup>He turned out to be the best medium for the heat transfer. On the one hand it is more expensive than <sup>4</sup>He, but on the other side, <sup>3</sup>He has a higher heat flux and less <sup>3</sup>He is necessary than for <sup>4</sup>He.

#### 3.2 Basic physical principles

In the previous section we have seen which coolant is the best, now we want to understand the basic physical principles of a cryostat. Therefore lets define some basic properties, Ref. [6] defined the Enthalpy of a system as

$$H = U + pV \tag{12}$$



Figure 16: [6] The principle of staedy flow in a closed system.

where U is the internal energy, p the pressure and V the volume. Since the internal energy describes the temperature of a system, we can say that the enthalpy indicates the heat of a thermodynamic system. The enthalpy can be related to the respective mass, what is called the specific enthalpy, which is given as

$$h = \frac{H}{m} \tag{13}$$

where m is the mass. So the specific enthalpy describes the heat of a specific mass.

#### Steady flow

Let us begin, the first law of thermodynamics is one of these basic principles, it says that in a closed system, the internal energy is always the same [6]. In other words, in closed systems, no energy is produced and none is absorbed. Therefore, we can say the Enthalpy is constant over the whole system [11], more precisely the whole heat that comes in is equal to the total heat that is brought out. Consequently, due to Eq. (13) the specific Enthalpy is constant as well. Now, we want to apply the first law of thermodynamics to a system of steady flow, a small flow system where the flow properties like velocity and mass flow will remain the same [33]. In this sense, the steady flow can be understood by Fig. 16, here the fluid enters with an enthalpy  $H_1$  and leaves with an enthalpy  $H_2$ , while the fluid is in the system, work  $\dot{W}$  is done on the system and heat  $\dot{Q}$  leaves the system. Note that in this assumption, the kinetic and potential energy of the flow is neglected. So if we now separate between in- and output we get.

$$H_1 + \dot{W} = H_2 + \dot{Q} \tag{14}$$

In summation, we can say that every kind of energy that comes in the system is equal to the energy that leaves the system.



Figure 17: The first part, a) shows he schematically function of a JT expansion, where the red dots represent helium particles. b) depicts the icon of a JT valve. [6]

#### Heat transfer to a cooler system

Another topic refers to the constant heat load from the UCN guide to the <sup>3</sup>He cooling bath. The heat supply to the <sup>3</sup>He will cause the <sup>3</sup>He to boil, therefore the Helium will be vaporized and be no longer available for cooling. Thus, it is necessary to replace all the gone liquid Helium in order to keep the cooling for the UCN guide.

In terms of the conceptional design report (CDR) [12], the <sup>3</sup>He refill rate can be obtained via

$$\dot{m} = \frac{\dot{Q}}{\Delta h} \tag{15}$$

where  $\dot{m}$  is the refill of <sup>3</sup>He,  $\dot{Q}$  is the heat of evaporation and  $\Delta h$  is the change in specific Enthalpy of the <sup>3</sup>He during the phase transition from liquid to vapor due to the boiling. The heat of evaporation equals the heat that needs to be transferred out, which is assumed to be 10 W. In other words, all the energy that comes from the UCN guide is used to evaporate the liquid Helium in the cooling pot. For the enthalpy change we need to consider the liquefaction process of the helium.

#### Joule-Thomson expansion

The liquefaction of helium is obtained by a expansion of the medium by a Joule-Thomson (JT) valve. Reference [6] describes this way of refrigeration, in this process is the pressure of the helium suddenly reduced so that the whole medium expands and a temperature drop is effected. In detail, the medium is flowing, under isenthalpic conditions, through a JT valve and gets in an irreversible process either heated or cooled. In Fig. 17 a) the cooling process is illustrated, here present red dots the helium particles that are flowing through a schematically JT valve. After the particles passed the valve, they expand over the space because the pressure is lower than before. The internal energy of the particles decreases by this expansion, what causes the temperature of each particle to decrease. However, whether the temperature increases or decreases, by using JT valve, depends on the JT coefficient  $\mu$  which is defined as

$$\mu = \left(\frac{\delta T}{\delta p}\right)_{\rm h},\tag{16}$$



Figure 18: The illustration shows the black inversion curve of <sup>4</sup>He and in green an example plot for constant Enthalpy. At the inversion curve  $\mu$  equals zero, the smaller part under the Graph indicate  $\mu$  smaller than zero and the remaining section stands for  $\mu$  greater than zero.

it describes the slope of a constant enthalpy curve. This slope must be positive for a cooling purpose and negative for warming. The inversion line, which is shown for <sup>4</sup>He in Figure 18, shows graphically the transition line, where  $\mu$  is changing its sign. The green line shows an example for an isenthalpic behavior and as mentioned before, in the section where the slope of the green line is positive, there is  $\mu$  smaller than zero. So we see, that the all values to the left of the transition line define the size of temperature and pressure that are needed, to reduce the temperature of the medium. So with the purpose to cool the Helium, it must be pre-cooled below this transition temperature so that cooling is obtained by the JT expansion.

The vapor pressure after this expansion is the saturated vapor pressure. In terms of [31] it is the pressure, at which as many liquid particles leave as come back in the liquid. More precisely, a molecule at a liquid surface is able to leave the fluid when its kinetic energy is high enough, in other words when the energy of a molecule reaches an amount equal to the heat of vaporization, so the molecule will get vaporized. In an open system the new vapor molecule would leave the system but in a closed container they can't leave and so it happens that some molecules get back in the liquid surface. So the pressure at which the leaving and incoming particles are the same, is called saturated vapor pressure. Moreover, this pressure depends strongly on the temperature of the liquid, because the larger the heat energy of the particles, the more liquid particles can escape, and so the vapor pressure rises. However, if we come back to the JT valve, it is important to mention that this valve does not provide ideal liquefaction, so always a small fraction of the helium is converted to vaporous helium. For the use at TRIUMF it follows, that during the cooling process already a small amount of helium is vaporized which won't be liquefied and so can't be used for the cooling of the UCN guide. For the receiving of the mass flow in Eq. (15) we need to consider this fact, because only the liquid can be used for the cooling purpose. Consequently, we need to consider the specific enthalpy change between the produced liquid and its vapor. Since the specific enthalpy is defined by the Enthalpy, see Eq. (13), we need to obtain the Enthalpy change between this points. Figure 19 make this point more clear, for the mass flow the enthalpy difference between the vaporized Helium, point "21", and the liquid Helium, point "11" must be obtained.

$$\Delta H = H_{2l} - H_{1l} \tag{17}$$

In order to obtain this enthalpy difference we take a closer look on the liquefaction and vaporization process of Fig. 19. Since the JT expansion is in a closed system and under isenthalpic conditions, we can say that the enthalpy remains the same. Therefore, the Enthalpy before the JT valve  $(H_0)$  is equal to the Enthalpy afterwards  $(H_{afterJT})$ .

$$H_0 = H_{\text{afterJT}} \tag{18}$$

As mentioned is the JT valve producing liquid and a fraction of vapor, so the total enthalpy after the expansion is the sum of both Enthalpies.

$$H_{\text{afterJT}} = H_{11} + H_{1v} \tag{19}$$

Consequently, the enthalpy at the beginning can be described as

$$H_0 = H_{11} + H_{1v}.$$
 (20)

Besides, the enthalpy at the end is described by

$$H_3 = H_{2l} + H_{1v} \tag{21}$$

The Enthalpy of the vapor at point "1v" can be received, by solving Eq. (20) for  $H_{1v}$ :

$$H_{1v} = H_0 - H_{1l}.$$
 (22)

So  $H_3$  can be written as

$$H_3 = H_{2l} + H_0 - H_{1l}.$$
(23)

Solved for  $H_{11}$ , Eq. (22) can be written as

$$H_{11} = H_{21} + H_0 - H_3. (24)$$

Now, by Eq. (24), Eq. (17) can be described as follows

$$\Delta H = H_{2l} - H_{2l} - H_0 + H_3. \tag{25}$$

Consequently,  $\Delta H$  is

$$\Delta H = H_3 - H_0. \tag{26}$$

With Eq. (13), the enthalpy can be written as

$$H = hm. (27)$$

Since the mass at point "3" and "0" is the same, Eq. (26) can be transformed to

$$\Delta h = h_3 - h_0. \tag{28}$$

So for the calculation of the mass flow, we can relate to the specific enthalpy difference between point "0" and "3".

Now let us consider the liquefaction rate of the JT valve. Equation (20) describes the separation of the Enthalpy at the valve, one part is liquefied  $(H_{11})$  and one part vaporized  $(H_{1v})$ . If we replace the enthalpy with the specific enthalpy of Eq. (13), the separation can be described in two different mass flows:

$$mh_0 = m_1 h_{11} + m_v h_{1v} \tag{29}$$

where m is the total helium mass,  $m_{\rm l}$  is the mass of liquefied helium and  $m_{\rm v}$  is the mass of vaporized helium. Since the total helium mass is the sum of liquefied and vaporized helium mass,  $m = m_{\rm l} + m_{\rm v}$ , we can define the flow of vaporized helium as

$$m_{\rm v} = m - m_{\rm l}.\tag{30}$$

Consequently, Eq. (29) can be expressed as

$$mh_0 = m_l h_{1l} + (m - m_l) h_{1v}.$$
(31)

Finally, for solving Eq. (31) for the liquefaction rate X, where  $X = \frac{m_1}{m}$ , we obtain

$$X = \frac{h_0 - h_{1v}}{h_{11} - h_{1v}}.$$
(32)

Lastly, to signal a JT valve in further graphics, the icon of Fig. 17 b) is used to indicate them.

#### Counterflow heat exchanger

The next principle uses the vaporized helium from the cooling baths to cool the inflow in previous steps. As mentioned, helium gets vaporized by the heat load from the UCN guide, this vapor is used to pre-cool the inflowing helium, illustrated in Fig. 21.

In terms of reference [32], the direction of the vapor flows is important to obtain the best possible cooling. In Fig. 20 are both possible types of flows, parallel and counter flow, depicted. For the case both guides flow in the same direction, we seen in Fig. 20 that the temperature difference between the two systems gets smaller and both are approaching to a common temperature. So the cooling gets smaller the closer the temperature of both flows gets, consequently, the received cooling in the end gets very poor. Next to it, for counter flowing fluids, Fig. 20 shows that, on the one hand the obtained cooling at the beginning is less. On the other hand, the temperature difference between the



Figure 19: A characteristic vaporization scheme of the cryostat. The Helium comes in at point "0" and passes the JT valve. The majority of the Helium gets liquefied and a fraction of it gets vaporized. Point "11" describes the liquefied Helium and "1v" the directly vaporized Helium. During operation, the heat input, indicated at the bottom, converts the liquid Helium (11) to vaporous Helium (21). Both vapor streams are depicted by point "3" together.



Figure 20: Parallel flow and counterflow are compared in terms of temperature exchange over time. The blue medium is the coolant which cools the red flow. [32]



Figure 21: The illustration shows the principle of a counterflow heat exchanger. The Helium is at the beginning in the liquid phase, than it flows through the pipe until it gets cooled and liquefied by a Joule-Thomson valve. After that the liquid is contained in a vessel in which it gets vaporised due to a heat input from underneath. The vapor gets pumped away through a pipe that is equipped with a part that is passed by the inlet pipe. In this section is the hotter incoming gas cooled by the cooler escaping gas. [30]



Figure 22: The schematic cryostat with focus on the IP Helium flow. On its way to the UCN guide, the IP Helium is cooled down by the two  $^{4}$ He baths and a JT valve. On the right hand side is the outlet of the

fluid is decreasing much slower. Since the warmer flux is already pre-cooled when the coolant enters the system, the coolant will warmth less fast. Therefore, the warmer side can be cooled over the whole time by a less heathen coolant, what provides a higher cooling power. In other words, the temperature difference between the flows remains high, so that more heat is exchanged.

In conclusion, the cooling by counter flow is much more efficient than the parallel flow. By comparing the parallel and counter flow diagrams of Fig. 20, the temperature of the warmer medium decreased to a much lower temperature than at the parallel flow. This can be achieved, because the coolant heats less fast .

#### 3.3 The two main helium systems

Now that we understand the basic principles of the cryostat, let's take a closer look at the two main helium systems.

#### **IP** Helium system

First of all, the IP helium is one of these main systems, it is purposed to cool the cold neutrons down to the ultra cold state via phonon downscattering. In order to prevent the IP helium from heating by this process, the heat is exchanged from the IP Helium system to a <sup>3</sup>He bath. Therefore, the liquid remains at its required temperature in the guide, thus, it isn't necessary to refill the guide with cold

medium. However, the IP helium needs to be cooled down before it is filled in the UCN guide, so it is pre-cooled by several steps, seen in Fig. 22. At first, the helium is lead through a <sup>4</sup>He pre-cooling system (not included in Fig. 22) before it enters the cooling path of Fig. 22, where it flows through parts of the <sup>3</sup>He pre-cooling system. Note that in this work the focus is on the cooling of the IP helium during operation, so the pre-cooling system of <sup>4</sup>He is neglected, the mentioned pre-cooling of <sup>3</sup>He will be the focus of a later consideration. Nevertheless, we see that that the IP helium is pre-cooled by two <sup>4</sup>He reservoirs and one JT valve between them. As discussed in the section before, is the JT valve the component, which liquefies the helium. Thereafter, the IP helium is at saturated vapor pressure and is cooled by the second <sup>4</sup>He reservoir to a temperature of 1 K, before it is filled in the UCN guide. After the production process, the IP helium is pumped out through the outlet.

In summary, the required cooling of the IP Helium is obtained leading it through two cooling baths of the <sup>3</sup>He pre-cooling system and a JT valve. However, the pre-cooling of IP helium has no influence on the <sup>3</sup>He pre-cooling system because the cooling of the IP helium takes place before UCN production begins.

### <sup>3</sup>He system

The second helium system we want to consider is the <sup>3</sup>He system (Fig. 23), which is purposed to cool the UCN guide. The heat is lead through the pipe to the <sup>3</sup>He bath, where it will cause the <sup>3</sup>He to boil. Thus, it is necessary to replace all the gone liquid Helium, to ensure further cooling of the UCN guide. The refill of <sup>3</sup>He requires a colling system, that provides <sup>3</sup>He at the required temperature for its cooling purpose. So the <sup>3</sup>He passes several different steps of refrigeration which can be distinguished in three different stages. Let us now look at all this different stages.

**Counter flow pre-cooling stage** At the beginning of the <sup>3</sup>He cooling system is a counterflow HEX stage, where the inflowing <sup>3</sup>He gets cooled by three series-connected counter flow HEX. As introduced before, vaporized helium from the baths of a latter stage supply the HEX ,,the at lower the temperature of the incoming <sup>3</sup>He. This section deals with the construction of the three heat exchangers of this stage.

The very first HEX, seen in Fig. 24 a), includes three flow channels, one includes the inflowing <sup>3</sup>He and two are guiding counter flowing <sup>4</sup>He from cooling baths of a latter stage. After the <sup>3</sup>He passed the first HEX it comes to the second and third HEX, illustrated in Fig. 24 b), it differs to the construction of the first HEX, because here, a connection to one of the two thermal shields is realized at each HEX. In other words, the second HEX is connected to warmer thermal shield and the third HEX is related to the colder thermal shield. The remaining built of theses two HEX is identical to the first HEX, they include next to the <sup>3</sup>He guide also the two upstreams of <sup>4</sup>He. For the sake of clarity, the thermal shields protect the system from outer heat radiation that could warm the whole system and therefore harm the whole refrigeration system. Note that on further consideration, the influence of the thermal shields is not taken into account.

<sup>4</sup>He pre-cooling stage After the first pre-cooling stage, the <sup>3</sup>He comes to the <sup>4</sup>He pre-cooling section, where it passes two <sup>4</sup>He baths and one counterflow HEX between the baths.

In a closer view, the <sup>4</sup>He refrigeration system is based on two connected pots, the 4K and 1K bath, of liquid <sup>4</sup>He where the <sup>3</sup>He flow is lead through. Figure 25 a) illustrates the way of the <sup>3</sup>He through the pots. The heat load of the <sup>3</sup>He to the helium in the baths, causes it to boil what requires a constant



Figure 23: The illustration displays the schematic construction of the whole <sup>3</sup>He precooling system which is built out of the three refrigeration stages. The red arrows shows the way of the <sup>3</sup>He from its way from the top to the <sup>3</sup>He bath and the way out out again. On the right hand side, the design shows the outflow of <sup>3</sup>He when it is evaporated during the cooling process of the UCN guide. [12]



Figure 24: a) depicts the schematic built of the very first counterflow HEX in the cryostat. b) shows the construction of the second and third HEX with the implement of a thermal shield. In this counterflow HEX are two <sup>4</sup>He upstreams cooling the one <sup>3</sup>He downstream and the thermal shield.



Figure 25: The part a) shows the flow of <sup>3</sup>He through the <sup>3</sup>He pots under neglecting of the <sup>4</sup>He flow. The pipes in each pot are spirally designed. Secondly, b) depicts the flow of liquid and gaseous <sup>4</sup>He, it enters in the liquid phase in the upper left corner and a portion of it continues flowing down to the second pot. In both baths gets the Helium vaporized and leaves each pot in the gaseous phase through the respective outlet on the top. In this second picture is the flow of the <sup>3</sup>He neglected. The real design is a combination of both plans.[12]


Figure 26: The scheme displays the counterflow HEX which is implemented between the two <sup>4</sup>He baths. The different colored arrows point the direction and medium of the flow. [12]



Figure 27: The figure shows the simplified second cooling stage with the counter flow HEX between the two  ${}^{4}$ He pots. [12]

refill of  ${}^{4}$ He as explained in Section 3.2.

From the beginning, by passing the first pot, which is at a temperature of 4 K, the <sup>3</sup>He get cooled down to almost the same temperature as the liquid in the pot. Like Fig. 25 suggests, the second cooling bath is provided with liquid helium by the 4K bath, since the <sup>3</sup>He is already cooled to almost 4 K, the second bath must be at a lower temperature. The cooling of <sup>4</sup>He between the reservoirs is obtained by a JT valve. The JT valve cool the <sup>4</sup> He down to a temperature of 1 K, before it is collected in the second bath. Analogous to the bath before, the heat load from the <sup>3</sup>He causes the helium in the bath to boil off. This vapor is used to cool the <sup>3</sup>He before it even reached the second cooling pot in a counterflow HEX. Moreover, the <sup>4</sup>He vapor supports to cool the <sup>4</sup>He between the two refrigeration baths as well. Thus, the counterflow HEX between the two cooling pots uses the rising <sup>4</sup>He vapor cool the incoming <sup>3</sup>He and the incoming <sup>4</sup>He, as schematically depicted in Fig. 26. Afterwards is this boil off and the boil off of the 4K bath used, for the cooling in the three counterflow HEX of the HEX stage, as shown in Fig. 24.

In summation, the <sup>3</sup>He is cooled at first by the 4K bath, then by counter flowing <sup>4</sup>He vapor and finally by the 1K bath. Figure 27 is showing the whole second cooling stage with the direction of the <sup>3</sup>He flow through the different components.

**JT cooling stage** At last, the <sup>3</sup>He obtains it final cooling in the JT cooling stage. Here is a JT valve the final element of the <sup>3</sup>He cooling system. The whole cooling stage is completed by a counterflow HEX before that JT valve, the HEX uses the vaporized Helium from the <sup>3</sup>He bath underneath to cool the Helium inlet right before the JT expansion. So this stage is comparable to Fig. 21.

## 3.4 Calculations of the cryostat

After we understand the built of the cryostat, we can look at some detailed calculations of the individual cooling capacities. Therefore, to obtain the heat transfer of each refrigeration step, the cooling power at each bath and HEX needs to be considered. For the sake of clarity during the calculation, we number each HEX before, as can be seen in Fig. 28 a). Figure 28 a) also shows the assumed temperature distribution of the helium through the cryostat, which was obtained by collaborators in Japan [12]. For the temperature increase of the counter flowing paths in a HEX, we assume that the <sup>4</sup>He temperature increases to the temperature of the <sup>3</sup>He at the beginning of the specific HEX. For instance, at HEX5, the <sup>3</sup>He enters at 80 K, than the the counter flowing <sup>4</sup>He heats in this HEX up to 80 K. Note that temperature changes of the helium that can occur while medium flows from one cooling step to another will be neglected in this work.

Before we start, lets clear some general topics. Since the mass flow in some pipes is generate by a pump, which pushes frequently a constant Helium volume through the pipe, we want to relate the helium flow to the required volume flow at the pump. This volume flux is given by

$$\dot{V} = \frac{\dot{m}}{\rho} \tag{33}$$

where  $\dot{m}$  is the mass flow and  $\rho$  the density of the vapor at the pump. For the volume flow we want to differ between system which need to be pumped on, the <sup>3</sup>He outlet and the 1K bath outlet, and system which don't need to be pumped on, the <sup>4</sup>He inlet and the 4K bath outlet. The volume flow of the pumping systems is stated in  $\frac{m^3}{h}$  and the volume flow of the other systems is stated referring their helium consumption in  $\frac{L}{h}$ . So the 4K bath inlet, which is equal to the total <sup>4</sup>He refill, and the 4K bath outlet are given in  $\frac{L}{h}$  and the outlet streams of the 1K bath and the <sup>3</sup>He bath are stated in  $\frac{m^3}{s}$ . In this sense, it is important to mention that the volume flux at the pump needs to be calculated at ambient temperature, due to the operation temperature of the pumps. This means, that the density of the helium is smaller than at the helium bath and according Eq (33), this effects a higher volume flux. On the other hand, the liquid consumption in the 4K pot is, due to the operation temperature, calculated at 4 K what means the density of the helium is higher. So it seems like the liquid consumption is much lower than the volume flow at the pump, but as mentioned, this phenomenon occurs only due to the density difference. Lastly, the following sections explain the proportion in which the respective values have been increased, this proportion always refers to the increase compared to the unchanged cryostat design with the assumed temperature distribution, listed in Table 10

### Performance of the HEX

Now we want to consider the heat performance of each flow in the HEX. The heat, every counter flowing path can take out and the heat that need to be brought out of the <sup>3</sup>He, can be calculated by changing Eq. (15) to



Figure 28: Both parts illustrates the cryostat with numbered points after each refrigeration step. The points are numbered from beginning to end in ascending order and each one represent different properties of the Helium in the pipe. Section A) refers to the <sup>3</sup>He system and section b) to the <sup>4</sup>He system. The latter system has two outlets and therefore, the <sup>4</sup>He system is separated in the warmer outlet, path "A", and the colder outlet, path "B".

$$\dot{Q} = \dot{m}\Delta h \tag{34}$$

where  $\Delta h$  is the change in specific enthalpy of the helium at the particular HEX and  $\dot{m}$  the repective helium flow rate. The mass flow can be obtained by Eq. (15), this mass flow equals the flow of helium at each location of the belonging pipe. For the change in specific enthalpy at each HEX we need to differ between flow system and its properties. Therefore, lets obtain the helium properties for the <sup>3</sup>He and <sup>4</sup>He system.

In this context are the necessary helium properties, namely Enthalpy, pressure and density, are obtained by HePak [22] and He3Pak [23]. Additionally, for the sake of clarity, the fridge is separated in different points, seen in Fig. 28, which help describing the helium properties at each location. Figure 28 is divided in the <sup>3</sup>He circle and the <sup>4</sup>He circle. The special thing about the <sup>4</sup>He is, because its two outlets it is separated in the warmer outlet "A" and the colder outlet "B". As mentioned before is the pre-cooling of the IP Helium in this context negligible.

## Properties of <sup>3</sup>He

Let us begin with the <sup>3</sup>He flow, the flow of the helium can be followed in Fig. 28, at the beginning <sup>3</sup>He enters at ambient temperature and half of atmospheric pressure the duct. Table 2 shows the obtained <sup>3</sup>He properties for the following steps. Referring to this, the pressure is assumed to be constant until the <sup>3</sup>He reaches the JT valve. After that the pressure in the duct is decreased to the saturated vapor pressure. Since the temperature drops along the system, the specific enthalpy continues to decrease from step to step. However, the sudden enormous decrease of the specific enthalpy at point 6 to 7,

Table 2: The heat properties of <sup>3</sup>He at every point of the cryostat are shown. The points refer to section a) of Fig. 28 and the state indicates the phase of the <sup>3</sup>He where "V" means vaporous and "L" means liquid.

	$^{3}\mathrm{He}$		
T [K]	p [Pa]	$h\left[\frac{J}{kg}\right]$	State
293	50662.5	2025831	V
80	50662.5	558162	V
10	50662.5	75006	V
6	50662.5	46686	V
4.2	50662.5	33359	V
3.2	50662.5	25157	V
1.6	50662.5	2649	$\mathbf{L}$
1	50662.5	1645	$\mathbf{L}$
0.8	377.75	780	$\mathbf{L}$
0.8	377.75	11985	V
1.6	377.75	17598	V
	T [K] 293 80 10 6 4.2 3.2 1.6 1 0.8 0.8 1.6	<sup>3</sup> He T [K] p [Pa] 293 50662.5 80 50662.5 10 50662.5 6 50662.5 4.2 50662.5 3.2 50662.5 1.6 50662.5 1 50662.5 1 50662.5 0.8 377.75 0.8 377.75 1.6 377.75	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

Table 3: The heat properties of <sup>4</sup>He in terms of outlet "A" are listed. The property points refer to section b) of Fig. 28 and the state indicates the phase of the <sup>4</sup>He where "V" means vaporous and "L" means liquid.

$^{4}$ He - outlet "A"						
Point	T [K]	p [Pa]	$h\left[\frac{J}{kg}\right]$	State		
1	4.2	120000	9933	L		
A2	4.2	120000	30743	V		
A3	10	120000	65177	V		
A4	80	120000	431017	V		
A5	293	120000	1537228	V		

can't be related to a temperature decrease only. Consequently, the low specific enthalpy must mean that between this points, in the 1K bath, the phase of the <sup>3</sup>He changes from gaseous to liquid. Next to it, as a result of the heat input at HEX1, the liquid <sup>3</sup>He vaporizes in the pot, which means on the one hand, the temperature stays constant and on the other hand, the specific enthalpy increases, what is indicated by point 9 and 10.

The flow of <sup>3</sup>He is obtained by using Eq. (15) where we have seen, that the change in specific enthalpy between point 8 and 10 in combination with the heat input of 10 W can be used. Consequently, the mass flow of <sup>3</sup>He results in 0.97  $\frac{g}{s}$ . The corresponding volume flow, at the pump, according Eq. 33 results in 7442 $\frac{m^3}{h}$ . Subsequently, the heat every HEX an <sup>4</sup>He bath takes out of the <sup>3</sup>He can be obtained by Eq. (34). The results are depicted in Table 6, 8 and 9 and will be discusses in a later section.

## Properties of <sup>4</sup>He

Now let us discuss the received properties of the <sup>4</sup>He, in Fig. 28 we see that it is filled in at point "1", where it is at a temperature of 4.2 K and a pressure of about 120000 Pa. The helium that gets vaporized in the 4K bath and leaves through outlet "A" is always at the same pressure from the beginning. The specific enthalpy rises due to the increasing temperature of the flow in the counter flow HEX. On the other hand, the <sup>4</sup> He that leaves the 4K bath in the direction of the 1K bath, gets

Table 4: The heat properties of <sup>4</sup>He referring the outlet "B" are shown. The property points refer to section b) of Fig. 28 and the state indicates the phase of the <sup>4</sup>He where "V" means vaporous and "L" means liquid.

$^{4}$ He - outlet "B"						
Point	T [K]	p [Pa]	$h\left[\frac{J}{kg}\right]$	State		
1	4.2	120000	9936	L		
B2	4.2	120000	9933	$\mathbf{L}$		
B3	3.2	120000	6258	$\mathbf{L}$		
B4	1.6	746.4	392	$\mathbf{L}$		
B5	1.6	746.4	23321	V		
B6	4.2	746.4	37047	V		
B7	10	746.4	67191	V		
B8	80	746.4	430720	V		
B9	293	746.4	1536843	V		

Table 5: The list refers to the cooling power of the two  ${}^{4}\text{He}$  baths in order to obtain the assumed  ${}^{3}\text{He}$  temperature after the pot.

Bath	$Q_{^{3}He}[W]$
$4\mathrm{K}$	12.9
$1\mathrm{K}$	21.8

expanded by a JT value so that the helium is after point "B3" at saturated vapor pressure. Since the <sup>4</sup>He in this path gets further cooled in the first steps, the specific enthalpy decreases. Thereafter, the specific enthalpy increases suddenly from point "B4" to "B5" due to the vaporization of the <sup>4</sup>He caused by heat load from the <sup>3</sup>He.

Now we want to obtain the <sup>4</sup> flow in this system, because of the two existing outlets, "A" and "B" (Fig. 28), the <sup>4</sup>He flow is distinguished in the flow of the inlet, the outlet flow of the 4K bath and the outlet flow of the 1K bath. The inlet flow is defined as the sum of the outlet flows from the two baths.

$$\dot{m_{\text{inlet}}} = m_{\text{outlet,A}} + m_{\text{outlet,B}} \tag{35}$$

To obtain the boil of in the <sup>4</sup>He baths, we need to consider the heat flows from the <sup>3</sup>He to each of the baths. This heat supply is calculated referring Eq. (34) where the <sup>3</sup>He flow and the enthalpy change of the <sup>3</sup>He is taken in account. The result of the heat loads to the two <sup>4</sup>He bath is recorded in Table 5. For the boil off rate, we use Eq. (15), where we use the obtained heat input and the associated change in specific enthalpy of the <sup>4</sup>He from the liquid to the gaseous state at each bath. Regarding Table 3, the change in specific enthalpy of the <sup>4</sup>He in outlet "A" is the difference between the liquid point 1 and vaporous point A2. On the other hand, the outlet "B" includes a JT valve and as known, the specific enthalpy difference between the state before the JT valve and after vaporization can be used in this calculation, like indicated in Eq (28). Therefore, the Enthalpy difference between point B3 and B5, of Table 4, need to be applied in the calculation of the mass flow.

In summation, the <sup>4</sup>He flow in outlet "A" results in 0.62  $\frac{g}{s}$  what equals an consumption of 17.6 $\frac{L}{h}$  in the pot and the flow in outlet "B" is 1.28  $\frac{g}{s}$ , which corresponds to a pumping volume of  $3745\frac{m^3}{h}$ . Hence, the total inlet flow of <sup>4</sup>He is 1.90  $\frac{g}{s}$  or  $53.9\frac{L}{h}$ . Now, the cooling power of each counterflow HEX can be calculated by Eq. (34). The results are shown in Table 6, 8 and 9, which will be discussed in the next section.



Figure 29: The schematic built of HEX4, HEX5 and HEX 6, with the relating heat flows. The origin of flow "A" and "B" are illustrated in Fig. 28. The thermal shields of HEX4 and HEX5 are neglected.

Table 6: Heat in- and output of the counterflow HEX4, HEX5 and HEX6. The inflowing <sup>3</sup>He is refrigerated by the two different outlets of <sup>4</sup>He. The last column indicates the total energy balance at the certain HEX. A positive value indicate heat input and a negative value heat output.

HEX	$Q_{3He}[W]$	$Q_A[W]$	$Q_B[W]$	$Q_{total}[W]$
4	27.4	-21.3	-38.5	-32.4
5	467	-227	-464	-224
6	1419	-685	-1412	-678

#### **Resulting performances**

After obtaining the heat input by the <sup>3</sup>He path and the heat output by outlet "A" and "B", compare Fig. 29, at each HEX, we want to discuss the combined performance of these three paths for each HEX. The influence of thermal shields is neglected in this observations.

Firstly, due to an similar built of HEX4, HEX5 and HEX6, comparable to Fig. 24, are the performance of this HEX compared in Table 6. The table compares the three refrigeration steps according the needed cooling of the <sup>3</sup>He in order to cool it down to the assumed temperature and it lists the cooling power provided by each of the two <sup>4</sup>He counterflow pipes. Moreover, the total energy balance of each HEX is summarized. When we look at required cooling powers it is noticeable that HEX6 need the highest required cooling energy, it is about three times bigger than that of HEX5. The needed cooling of HEX4 is with respect to the other two HEX very small. This can be explained by the small temperature change of the <sup>3</sup>He at HEX4 compared to the temperature changes at HEX5 and HEX6. The same behavior applies to the obtained cooling by the two upstreams of <sup>4</sup>He. Here is the received cooling power at HEX5 and HEX6 very large, whereas the cooling power of HEX4 is much less due to its smaller temperature change. Moreover, while examining the data of Table 6, we notice that the necessary cooling for the <sup>3</sup>He can be achieved almost exclusively through the outlet "B". Because of that, outlet "A" could be removed from the counter flow HEX constructions under a small increase of the cooling power of upstream "B", as a result, the HEX could be easier designed. So the question is, how can the cooling of outlet "B" be improved. This can be achieved by a higher vapor flow, so the real

Table 7: Heat in- and output of the counterflow HEX4, HEX5 and HEX6 for only one counter flowing path, outlet B of Fig. 29. To obtain the required cooling, either the temperature at point "B3" is increased or the temperature of point "B4" is decreased, compare points to Fig 28 b).  $Q_{3\text{He}}$  indicates the necessary cooling of the <sup>3</sup>He and the last column indicates the total energy balance at the certain HEX. A positive value indicate heat input and a negative value heat output.

HEX	$T_{\rm B3}~[{\rm K}]$	$T_{\rm B4}$ [K]	$Q_{3He}[W]$	$Q_{\rm B}[W]$	$Q_{total}[W]$
4			27.4	-39.1	-11.7
5	3.3	1.6	467	-472	-5
6			1419	-1435	-16
4			27.4	-39.5	12.1
5	3.2	1.5	467	-476	-9
6			1419	-1450	-31

Table 8: Heat in- and output of counterflow HEX3. Here are the inflow of  ${}^{3}\text{He}$  and  ${}^{4}\text{He}$  are cooled by vaporized  ${}^{4}\text{He}$  of the 1K pot.

question is how can we boil off more liquid from the 1K bath. This can be seen in Eq. (15, there the mass flow gets bigger for a greater heat supply or a decreasing change in the specific enthalpy. Since the heat load from the <sup>3</sup>He stays the same, we must focus on a decreasing enthalpy change. Because of the JT cooling at the 1K bath, we can use Eq. (28) to describe the change in specific enthalpy. Equation. (28) gives us two possibilities, the first one is to decrease the specific enthalpy after the expansion and the second is to increase the specific enthalpy before the the JT valve. Since the specific enthalpy is the enthalpy over the mass (Eq. (13)) and the enthalpy describes the internal energy or in other words, the temperature of a system (Eq. (12)), we can follow that either the temperature of the 1K bath must be decreased or the temperature of liquid before the JT valve must be increased. Table 7 considers this two cases, here is at first the temperature before the JT valve (point "B3") increased by 0.1 K and then the performance of outlet "B" compared with the required cooling of the <sup>3</sup>He. The same procedure is repeated for the 1K bath, with the difference that there the temperature of the 1K bath (point "B4") is decreased by 0.1 K. The results indicate that a temperature increase or respective decrease of 0.1 K at the certain location is enough to obtain the required cooling. Consequently, for this assumptions can the outflowing path "A" be removed from the HEX4, HEX5 and HEX6. Nevertheless, in order to decide which of these two options is appropriate, it should be investigated how the different temperatures at the different points can be achieved and what effects this changes will have on the rest of the cryostat. Note that in this work, the performance of the cryostat is discussed, so a discussion about the construction of the different components of the cryostat it has to be a subject of additional studies.

In summation, the total heat balance for every HEX shows for that the assumed cooling at every HEX can be obtained. Since the necessary cooling for the helium can be achieved almost exclusively through the outlet "B", outlet "A" could be removed from the counter flow HEX constructions, so that an the HEX could be easier designed. However, since a poor performance of HEX4 would increase the heat load to later cooling stages and therefore the change all the assumptions, we will discuss a poor performance of HEX4 in a later section.

Next, Table 8 depicts the cooling performance of counterflow HEX3. In contrast to the first mentioned

Table 9: Heat in- and output of counterflow HEX2. The <sup>3</sup>He that flows in gets cooled by the vapor that gets pumped away.

Table 10: This is the reference table for the following assumptions. The mass flow and its respectively volume flow for an unchanged cryostat are listed. The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate  $(\frac{m^3}{h})$ , and the non-pumping systems, where the volume flow is stated in the liquid consumption  $(\frac{L}{h})$ .

Flow system	$\dot{m}[\frac{g}{s}]$	$\dot{V}$
$^{3}\mathrm{He}$	0.97	$7442 \ \frac{m^3}{h}$
4K outlet	0.62	$17.6 \frac{L}{h}$
1K outlet	1.30	$3808 \ \frac{m^3}{h}$
$^{4}$ He inlet	1.92	54.5 $\frac{L}{h}$

HEX are here to inflows that need to be cooled and only the one <sup>4</sup>He path that offers the cooling like the schematic built in Fig. 26. More precisely, the flow of <sup>3</sup>He as well as the <sup>4</sup>He before it reaches the JT valve needs to be cooled. The cooling is provided by the drain of evaporated <sup>4</sup>He from the 1K pot. The obtained refrigeration by this drain is higher than the needed coolings for the two flows together, so the required cooling of the baths can be received. Due to the less cooling of the <sup>3</sup>He, with only 8 W, the question, whether this HEX is necessary to the cryostat or not, will be analyzed later.

After that let us compare the heat statistics of HEX2 which is listed in Table 9. Here is the pre-cooling of the <sup>3</sup>He provided by the <sup>3</sup>He vapor produced at HEX1. In this case, the temperature of the incoming Helium needs to be reduced of 0.6 K what equals a cooling power of 0.97 W. The obtained cooling is about 5.5 W and so the cooling can be achieved. As with HEX3 is the required cooling of the <sup>3</sup>He very small, here the amount is even smaller and so the importance of HEX2 seems to be very less, therefore the use of HEX2 will be discussed in the next section.

After all we can conclude, with the assumptions that were made, every HEX is able to achieve the required cooling. Furthermore, the small required cooling by HEX2 and HEX3, is raising the question of the need for these two HEX. Additionally, a poor performance of HEX4 would increase the heat load to other cooling stages. For this reason, the importance of the mentioned HEX will be discussed next.

#### Importance of the HEX

Now, with the obtained performances, let us discuss whether HEX2 and HEX3 are necessary at all and additionally we want to imagine the case of a poor performance of HEX4. For every assumption we want to refer to properties of the basic system, listed in Table 10, where all HEX are included and each is assumed to work properly. An important point in this section is, that the obtained volume flow is distinguished between the pumping systems (outlet of the 1K and <sup>3</sup>He bath), where the volume flow is in the pumping rate  $(\frac{m^3}{h})$  stated, and the non-pumping systems (<sup>4</sup>He inlet and 1K bath outlet), where the volume flow is in the liquid consumption  $(\frac{L}{h})$  stated.

**HEX2** Firstly, according Table 9 HEX2 must provide the lowest cooling capacity with less than 1 W. Therefore, the question is whether HEX2 can be removed or not. In other words, what effects will the

Table 11: The mass flow and its respectively volume flow of all affected flow systems are shown for a removal of HEX2. The proportion by which the flows increased compared to the design with HEX2 (Table 10) is listed. The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate  $(\frac{m^3}{h})$ , and the non-pumping systems, where the volume flow is stated in the liquid consumption  $(\frac{L}{h})$ .

Flow system	$\dot{m}[\frac{g}{s}]$	$\dot{V}$	Proportion [%]
<sup>3</sup> He	1.1	$8240 \ \frac{m^3}{h}$	11
4K outlet	0.7	19.5 $\frac{L}{h}$	11
1K outlet	1.4	$4150 \ \frac{m^3}{h}$	9
$^{4}$ He inlet	2.1	59.7 $\frac{L}{h}$	10

lack of HEX2 have to the cryostat. If mentioned HEX is neglected, the liquid <sup>3</sup>He is not pre-cooled when it reach the JT valve, so it still has the temperature it has when it left the 1K bath. The higher temperature of the medium at the JT valve means the Enthalpy of the Helium is higher than before at this point. Because of that, the efficiency of the JT valve decreases according Eq. (26). In other words, when the same amount of <sup>3</sup>He is provided to the valve, less liquid than before is produced. Consequently, a higher <sup>3</sup>He flow is required to obtain the same amount of liquid than before and therefore the pump has to push more  ${}^{3}$ He through the pipe. Moreover, in terms of Eq. 34 a greater amount of <sup>3</sup>He pushed through the system, effects a greater heat input to the baths and HEX. So each pre-cooling step has to deliver a greater amount of cooling, in order to cool the <sup>3</sup>He in the pipe to the wanted temperature. So the removal of HEX2 would affect every cooling pot and every HEX. In this sense, if the heat input to the <sup>4</sup>He baths increases, than more liquid is vaporized and therefore a larger amount of refill is required. In other words, a higher inlet flow of <sup>4</sup>He is required as well. Another effect of the bigger amount of vaporized Helium is that the pumps need to move more vaporized Helium than before. At last, the increasing heat supply to the counter flow HEX can be neglected, because as mentioned increases the rising  ${}^{3}$ He flow the vaporization rate in the  ${}^{4}$ He baths which supply the counter flow HEX. So on the one hand each HEX has too cool more <sup>3</sup>He, but on the other hand each HEX has also more counter flowing vapor available for the cooling purpose.

In summary, the removal of HEX2 would increase the flow of <sup>3</sup>He and so the performance of the pump and each pre-cooling step as well. The cooling power of the HEX can be compensated by the increased vaporization rate in the cooling pots without any further cause. The increased vaporization rate in the <sup>4</sup>He bath, on the other hand, requires a larger refill in each pot and therefore also a larger performance of each pump. Table 11 examines the effect on each property, so the mass flow and respectively the volume flow at each system increases of 11 %. This has the biggest impact on the <sup>3</sup>He system, where the volume flow rises to  $8.2 * 10^3 \frac{m^3}{h}$ , due to the already high volume flow.

**HEX3** Secondly, HEX3 provides only about 8 W of cooling power, therefore it can be considered to be removed from the cryostat. So we want to consider what effects an removal of HEX3 has to the cryostat.

For the case that HEX3 is removed, the <sup>3</sup>He has a higher temperature when it enters the 1K bath because it isn't pre-cooled by HEX3. For the 1K pot it means that it has to cool the <sup>3</sup>He from 4.2 K down to 1.6 K instead of from 3.2 K down to 1.6 K. In other words, the heat input to the 1K pot is greater than before. Therefore more liquid <sup>4</sup>He gets evaporated and a larger refill of Helium is required. So the pumps have to pump more Helium in and out of the 1K bath. Another effect is, because of the larger amount of vapor, the counter flow HEX4, HEX5 and HEX6 are provided with more cooling

Table 12: The mass flow and its respectively volume flow of the <sup>4</sup>He system are shown for a removal of HEX3. The proportion indicates by which amount the flows increase compared to the design with HEX3 (Table 10). The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate  $(\frac{m^3}{h})$ , and the non-pumping systems, where the volume flow is stated in the liquid consumption  $(\frac{L}{h})$ .

Flow system	$\dot{m}[\frac{g}{s}]$	$\dot{V}$	Proportion $[\%]$
<sup>3</sup> He	0.97	$7442 \ \frac{m^3}{h}$	0
4K outlet	0.62	$17.6 \frac{L}{h}$	0
1K outlet	2.2	$6510 \ \frac{m^3}{h}$	71
$^{4}$ He inlet	2.8	$80.7 \frac{L}{h}$	48

power. Furthermore, HEX3 provides pre-cooling of <sup>4</sup>He before it reaches the JT valve and , similar to the removal of HEX2, if HEX3 is taken out the JT efficiency gets lower due to a higher Enthaply of the entering Helium. As in the case before, is less liquid Helium obtained by the same input to the JT valve. In order to compensate the decreasing JT efficiency, the refilling amount of <sup>4</sup>He has to get bigger as well.

In summation, the removal of HEX3 has two effects, at first is more <sup>4</sup>He evaporated because of the higher heat input by <sup>3</sup>He and in addition the JT efficiency decreases what decreases the received amount of liquid <sup>4</sup>He. So both reasons require a larger flow of <sup>4</sup>He what means the volume flow at the inlet and at the outlet of the 1K bath increase. Subsequently, the higher vapor flow effects that the counter flow HEX4, 5 and 6 have more vapor for their cooling purpose available.

In conclusion, removing HEX3 will affect only the <sup>4</sup>He system. The missing cooling of HEX3 can be compensated by the colder <sup>4</sup>He pot, but the larger heat input requires a larger refill of liquid <sup>4</sup>He. Moreover, because of no pre-cooling of the <sup>4</sup>He the JT efficiency decreases and a larger amount of <sup>4</sup>He is needed to obtain the same amount of liquid. Both consequences show that the removal of HEX3 increase the necessary refill quantity of <sup>4</sup>He and the volume flow at the outlet of 1K bath. In detail, according Table 12, the refill volume will increase from  $54\frac{L}{h}$  by 50 % to  $81\frac{L}{h}$  and the outlet volume flow at the 1K bath rise from  $3.7 * 10^3 \frac{m^3}{h}$  by 74 % to  $6.5 * 10^3 \frac{m^3}{h}$ .

**HEX2 and HEX3** Now let us discuss the removal of both, HEX2 and HEX3. In this assumption will, as mentioned above, a missing HEX2 cause an increase of the <sup>3</sup>He flow all over the cryostat what results in a heat load to the cooling baths what increases the vaporization rate in the two <sup>4</sup>He pots. In addition, the remove of HEX3 will, for known reasons, increase the <sup>4</sup>He consumption at the colder bath as well. So the remove of the two HEX is for the 4K pot equal to the case of excluding only HEX2. On the other hand, at the 1K bath are both removals are effecting a higher need in <sup>4</sup>He. In summation is the <sup>4</sup>He consumption at the 1K pot increased by the greater <sup>3</sup>He flow due to missing of HEX2, the greater <sup>3</sup>He temperature because of no pre-cooling by HEX3 and finally by the decreasing JT efficiency owing to the lack of <sup>4</sup>He cooling by HEX3. So the <sup>4</sup>He vaporization rate at the colder bath gets even higher than in the examples before. Simultaneously is with the higher amount of boil off the cooling performance of each counterflow HEX increased. As before, the HEX needs more cooling power to cool the increased <sup>3</sup>He inflow, which is generated by the increased amount of vapor. However, due to the three effects of an increasing vapor flow, in the 1K bath, each HEX will exceed its cooling performance even more. So not only the higher <sup>3</sup>He inflow is compensated, but also surpassed.

In total, under consideration of the calculated results of Table 13, the  ${}^{3}$ He flow and the outflow of the 4K bath are, as mentioned, equal to the flow rates when only HEX2 is removed. More conspicuous are

Table 13: The mass flow and its respectively volume flow of all affected flow systems are shown for a removal of HEX2 and HEX3. The proportion by which the flows increased compared to the design with both HEX ((Table 10)) is listed. The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate  $(\frac{m^3}{h})$ , and the non-pumping systems, where the volume flow is stated in the liquid consumption  $(\frac{L}{h})$ .

Flow system	$\dot{m}[\frac{g}{s}]$	$\dot{V}$	Proportion [%]
$^{3}\mathrm{He}$	1.1	$8240 \ \frac{m^3}{h}$	11
4K outlet	0.7	19.5 $\frac{L}{h}$	11
1K outlet	2.5	$7210 \ \frac{m^3}{h}$	89
$^{4}$ He inlet	3.1	$89.3 \frac{L}{h}$	64

Table 14: The mass flow and its respectively volume flow of the <sup>4</sup>He system are shown for a removal of HEX4. The proportion indicates by which amount the flows increase, compared to the design including HEX4 (Table 10). The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate  $\left(\frac{m^3}{h}\right)$ , and the non-pumping systems, where the volume flow is stated in the liquid consumption  $\left(\frac{L}{h}\right)$ .

Flow system	$\dot{m}[\frac{g}{s}]$	$\dot{V}$	Proportion $[\%]$
$^{3}\mathrm{He}$	0.97	$7442 \ \frac{m^3}{h}$	0
4K outlet	1.94	55.0 $\frac{L}{h}$	212
1K outlet	1.30	$3808 \ \frac{m^3}{h}$	0
$^{4}$ He inlet	3.23	91.9 $\frac{L}{h}$	69

the flow rate of the 1K pot with an increase from  $3.7 * 10^3 \frac{L}{h}$  by 93 % to  $7.2 * 10^3 \frac{L}{h}$  and the flow rate of the <sup>4</sup>He inlet with a rise from  $54 \frac{L}{h}$  by 66 % to  $89 \frac{L}{h}$ . Nevertheless, the volume flow of the <sup>3</sup>He is still the most dominant of all it increases from  $7.4 * 10^3 \frac{m^3}{h}$  by 11 % to  $8.2 * 10^3 \frac{m^3}{h}$ .

**HEX4** Now we want to consider the performance of HEX4. As already mentioned, seems the obtained cooling capacity by this HEX very critical. So, we want to consider the possibility of a not enough refrigerated <sup>3</sup>He by HEX4. In other words, the case of <sup>3</sup>He entering the 4K bath at a higher temperature than expected.

So if HEX4 can't provide the required cooling power, then the next cooling step, the 4K bath, will have to compensate the missing amount of cooling. So the bath needs to cool the <sup>3</sup>He from a higher temperature or in other words the enthalpy change of the <sup>3</sup>He is bigger than before. In terms of Eq. (34), this means that because of the increasing heat input to the bath, a evaporation rate must be taken into account. The obtained heat load via Eq. (34) allows to calculate the <sup>4</sup>He consumption according Eq. (15). Figure 30 examines the heat load and the <sup>4</sup>He consumption over an rising HEX4 exit temperature, compare to Table 15. The HEX4 exit temperature of 6 K, in the table and plots, means that HEX4 works as expected and the exit temperature of 10 K equals the worst case for a missing HEX4. The effects of the worst case, a removal of HEX4, to the flow systems is listed in Table 14. As mention, the flow rate of the 4K bath increases heavily. It can be seen, that the heat load to the bath increases linearly what causes an linear increase of the <sup>4</sup>He consumption. The lack of HEX2 is in both plots added, as mentioned, the removal of HEX2 effect a higher flow of <sup>3</sup>He and so the heat load and Helium consumption increase is even higher than for the construction with HEX2.

All in all, for a poor performance of HEX4 the heat load to the 4K bath rises, in the worst case, to roughly 45 W. This heat input would require and <sup>4</sup>He usage at the inlet of about 57  $\frac{L}{h}$  for the



Figure 30: Both diagrams consider between two cases, the first (red line) is in respect to the cryostat with HEX2 and the second case (green line) is in terms of the cryostat with a removed HEX2. Graph a) shows the heat load to the 4K bath under an increasing HEX4 exit temperature. Part b) depicts the consumption rate of <sup>4</sup>He in the 4K bath under an rising HEX4 exit temperature.

Table 15: The heat input into the 4K bath and its affected volumetric flow are listed in terms of the <sup>3</sup>He temperature after leaving HEX4. The inlet consumption is stated for the construction with and without HEX2.

	With HEX2		Without HEX2	
HEX4 Exit temperature [K]	$Q_{4Kbath}[W]$	$\dot{V}_{in}[\frac{L}{h}]$	$Q_{4Kbath}[W]$	$\dot{V}_{in}[\frac{L}{h}]$
6	12.9	17.6	14.3	19.5
7	19.8	27.0	21.9	30.0
8	26.7	36.4	29.5	40.3
9	33.5	45.7	37.1	50.7
10	40.3	55.0	44.6	60.9

Table 16: The mass flow and its respectively volume flow of all affected flow systems are shown for a removal of HEX2, HEX3 and HEX4. Furthermore, the proportion by which the flows increased compared to the design with both HEX (Table 10) is listed. The volume flow differs between the pumping systems, where the volume flow is specified in the pumping rate  $(\frac{m^3}{h})$ , and the non-pumping systems, where the volume flow is stated in the liquid consumption  $(\frac{L}{h})$ .

Flow system	$\dot{m}[\frac{g}{s}]$	$\dot{V} * 10^6 [\frac{L}{h}]$	Proportion [%]
$^{3}\mathrm{He}$	1.1	$8240 \ \frac{m^3}{h}$	11
4K outlet	2.1	$60.9 \ \frac{L}{h}$	246
1K outlet	2.5	$7210 \ \frac{m^3}{h}$	89
$^{4}$ He inlet	4.6	$131 \frac{L}{h}$	140

construction with HEX2 and about 63  $\frac{L}{h}$  for a built without HEX2.

**Removal of HEX2, HEX3 and HEX4** At last, let us consider the case for a removal of HEX2 and HEX3 with a poor performance of HEX4. In this condition we want to assume the worst performance of HEX4, the case for a not working or removed HEX4.

Analogous to the case of removal of HEX2 and HEX3, the consumption of <sup>3</sup>He and <sup>4</sup>He will rise in addition comes the higher heat load to the 4K bath due to a not working HEX4. So the higher heat load to the bath will increase the <sup>4</sup>He consumption even more.

In summation, for the worst assumed case, the outlet at the 1K bath is the part with the highest increase by 246 %, the liquid consumption rises to  $60\frac{L}{h}$  due to the big heat load brought to the bath by the <sup>3</sup>He. Thereafter, the <sup>4</sup>He inlet rises from 53.9  $\frac{L}{h}$  by 143 % to 130  $\frac{L}{h}$ . The 1K outlet and the <sup>3</sup> He flow stay the same as in the case for the removal of only HEX2 and HEX3 because the poor performance only effects the <sup>4</sup>He cooling bath.

In conclusion, we looked at the whole cryostat with its assumptions and can say that the cryostat is able to deal with the 10 W heat load to HEX1. However, we noticed that the cryostat can be easier designed, for instance the counterflow HEX4, HEX5 and HEX6. Where one cooling path could be removed when the cooling power of the second one is slightly improved. Moreover, we understand that some of the counter flow HEX are negligible due to their low performance, namely HEX2 and HEX3. On the other hand, HEX4 is a very critical HEX due to its influence on the  ${}^{4}$ He consumption in the 4K bath. Finally, the data in Table 17 shows the causes of the removal of individual HEX. For the <sup>3</sup>He flow only matter the removal of HEX2, when this HEX is excluded the flow rises to 8240  $\frac{\text{m}^3}{\text{b}}$ . This increased <sup>3</sup>He flow increases all <sup>4</sup>He flows as well, since the more <sup>3</sup>He needs to be cooled. Secondly, the outlet flow of <sup>4</sup>He in the 4K bath is heavily dependent on the performance of HEX4. For the worst performance of this HEX, what equals the removal of it, the <sup>4</sup>He flow more than doubles. The removal of the remaining to HEX affects this flow very little or not at all. Furthermore, the <sup>4</sup>He flow through the 1K bath increases significant as soon as HEX3 is removed, because then the <sup>3</sup>He isn't pre-cooled and the heat load to the 1K bath rises, so that the missing of HEX3 alone can increase the required amount of <sup>4</sup>He by about 3000  $\frac{\text{m}^3}{\text{h}}$ . The inlet flow of <sup>4</sup>He reflects the combination of the two <sup>4</sup>He outlet flows, here is the worst case when both <sup>4</sup>He baths have no pre-cooling or in other words, when HEX3 and HEX4 are removed. Additionally, for an increased <sup>3</sup>He flow, due to the removal of HEX2, the inlet consumption rises to up to 131  $\frac{L}{h}$ .

Table 17: The volume flow of the individual flow systems for the general built with all HEX and the built with the removal of one or more HEX. The flow is distinguished between pumping systems, where the volume flow is specified in the pumping rate  $\left(\frac{m^3}{h}\right)$  stated, and the non-pumping systems, where the volume flow is stated in the liquid consumption  $\left(\frac{L}{h}\right)$ .

		. 10 .		
HEX	$V_{3He}[\frac{m^3}{h}]$	$V_{4Kbath}[\frac{L}{h}]$	$V_{1Kbath}[\frac{m^3}{h}]$	$V_{4Heinlet}[\frac{L}{h}]$
/	7442	17.6	3808	54.5
2	8240	19.5	4150	59.7
3	7442	17.6	6510	80.7
4	7442	55.0	3808	91.9
2 & 3	8240	19.5	7210	89.3
2 & 3 & 4	8240	60.9	7210	131

# 4 Pressure Drop

In the section of the cryostat design we talked about the boil off in the <sup>3</sup>He bath and the vapor volume that must be pumped by the pump. Back in this section, we assumed that the vapor pressure at the liquid surface of the bath is equal to the vapor pressure at the pump. This assumption idealizes the outflow, as the pressure decreases with increasing steam temperature. In this sense, we will estimate the pressure drop in the <sup>3</sup>He outlet, look at the factors of influence and the consequences of an occurring pressure drop. Because of these causes, the design of the outlet is discussed in terms of low pressure loss.

# 4.1 Design of the <sup>3</sup>He outlet

At first, we look at the design of the <sup>3</sup>He outlet system because, as we will see, the pressure drop strongly depends on the design of the outlet. The diameter is neglected in this section, an analysis of the diameter follows later. In the current design of the 3He outlet, which is depicted in Fig. 31, the flow starts in the pipe at the <sup>3</sup>He pot where the Helium gets vaporized and thereafter it moves through a straight pipe until it reaches the first bending. We want to call this part the first part of the outlet and as can be seen in the figure, three temperature clamps are fixed at the outside of this pipe section. The first clamp is at height of 0.35 m over the bath and at a temperature of 4 K. The next is 0.6 m over the bath with a temperature of 50 K and the third one is at a height of 2 m over the bath with a temperature of 300 K. After this straight part follows a 90°-bending, the second segment. This bending has a radius of 0.45 m and is followed from another straight part, part three, with a length of 3m. Section four is a second 90°-bending with a radius of the bending of 0.45 m. The last component, part five, is a straight part with a length of 0.5 m which is followed by the pump.

So basically is the outlet built of three straight parts and two 90°-bendings. In terms of this design, we want to consider the behavior of the pressure in these straight and bended parts.

## 4.2 Pressure drop in a straight pipe

Let's start with the straight pipes, in relation to [8], the pressure drop in these constructions can be calculated by the Darcy-Weisbach equation

$$\Delta p = 4f \frac{\Delta x}{D} \frac{\rho v^2}{2} \tag{36}$$



Figure 31: The figure shows schematically the outlet of the <sup>3</sup>He. It gets vaporized in the <sup>3</sup>He pot and on the way to the pump the vapor passes section 1 - 5 where section 2 and 4 are 90° bendings. Along section 1 are three temperature clamps fixed that heat the up flowing vapor.



Figure 32: The schematic laminar and the turbulent regime. [9]

where f is the friction factor, L is the length of the pipe, D is the diameter of the pipe, rho the density and v the velocity.

### Friction factor

In terms of Van Sciver [6], the friction factor f expresses the friction between the vapor and the surface of the duct. The calculation of this property is dependent on the kind of flow in the duct. There are two possible regimes, the laminar and the turbulent flow which are depicted in Fig. 32. Laminar flow is when the vapor on the inside is flowing parallel to the walls of the pipe. In this case, the friction is comparable low because it has less interaction with the walls. The friction factor of laminar flow is defined as

$$f = \frac{16}{Re} \tag{37}$$

where Re is the Reynolds number. Furthermore, turbulent flow is when vapor swirls through the pipe. This means that the gas is almost flowing randomly through the pipe and consequently, the gas particles hit the wall more often. So turbulent flow is expressed by a greater friction factor which is obtained by

$$f = \frac{0.079}{Re^{0.25}} \tag{38}$$

where Re is the Reynolds number.

#### **Reynolds** number

The size of this number is an indicator for the kind of flow. A Reynolds number less than 2000 indicates laminar flow and every value larger than that states a turbulent flow [7]. The Reynolds number is characterized

$$Re = \rho v \frac{D}{\mu} \tag{39}$$

where  $\rho$  is the density of the Helium, v is the velocity of the flux, D is the Diameter of the duct and  $\mu$  is the viscosity. The density and viscosity are properties of the Helium which can be obtained by He3Pak [22].

## Velocity

The velocity is given by

$$v = \frac{\dot{m}}{\rho A} \tag{40}$$

where  $\dot{m}$  is the mass flow,  $\rho$  is the density and A is the cross area of the flow which is described by

$$A = \pi r^2. \tag{41}$$

In summation, the pressure drop depends on the geometry of the pipe, the properties of the flow and the properties of the Helium. In terms of Helium properties, the density and the viscosity are properties of the Helium which will change with the pressure and the temperature through the pipe. Therefore, for an accurate calculation of the pressure drop, it is important to determine the temperature across the duct to obtain the right Helium properties. In this sense, the temperature change through the tube must be considered.

### Temperature change

Regarding reference [7] is the temperature change described via

$$\Delta T = \frac{Q}{\dot{m}c_{\rm p}} \tag{42}$$

where Q is the heat transfer from the warmer inside to the colder outside,  $\dot{m}$  is the vapor flow in the pipe and  $c_p$  is the specific heat at constant pressure.  $c_p$  can be used because for the small fraction of the pipe where the temperature change is obtained, it is assumed that the pressure stays constant. At known temperature this specific heat can be provided by He3Pak. Besides, the heat transfer out of a small piece of the duct is given by

$$Q = h\pi D\Delta x (T_{\text{wall}} - T_{\text{vapor}}).$$
(43)

where h is the heat transfer coefficient, D the diameter,  $\Delta x$  is the small fraction of the tube,  $T_{wall}$  is the temperature of the wall and  $T_{vapor}$  is the temperature of the vapor in the inside.

#### Heat transfer coefficient

The heat transfer coefficient h is given by

$$h = Nu\frac{k_{\rm v}}{D} \tag{44}$$

where Nu is the Nusselt number,  $k_v$  is the heat conductivity of the vapor and D is the Diameter of the pipe. The heat conductivity is provided by He3Pak. According to [10], the positioning of the pipe must be considered in this calculation, if the channel is in a horizontal position, the diameter in this

Section	L [m]	$\Delta T_{\rm max}  [{\rm K}]$	$\frac{\Delta I_{\text{max}}}{L} \left[\frac{\kappa}{m}\right]$
1a	0.35	3.2	9.14
$1\mathrm{b}$	0.6	46	76.7
1c	1.94	250	128.9
2	1.4	0	0
3	0.5	0	0
4	1.4	0	0
5	0.5	0	0

Table 18: The wall temperature increase at the different parts of the  ${}^{3}$ He outlet. The bended part, 2 and 4, are listed with their effective length. The sections are depicted in Fig. 31.

equation is replaced by the characteristic diameter. Otherwise, in a vertical position of the pipe, the normal diameter is used in a vertical position. The characteristic diameter can be calculated via

$$D_{\rm ch} = \frac{\pi}{2}D$$

#### Nusselt number

The Nusselt number depends on the previously mentioned types of flow. For laminar flow the Nusselt number equals 4 and for turbulent flow the Nusselt number is provided by the Dittus-Boelter correlation

$$Nu = 0.023 Re^{0.8} Pr^{0.3} \tag{45}$$

where Re is the Reynolds number and Pr is the Prandtl number. The Prandtl number can be computed by He3Pak.

#### Wall temperature

At last, to obtain the heat transfer of Eq. (43) the temperature of the wall must be estimated. Therefore, a linear behavior of the temperature from the beginning to the end is assumed.

$$T_{\text{wall}} = \Delta x \frac{\Delta T_{\text{max}}}{L} + T_0 \tag{46}$$

In the equation,  $\Delta x$  is the length at which the wall temperature is searched, L is the length of the guide,  $\Delta T_{max}$  is the entire change of wall temperature at a certain part of the pipe and  $T_0$  is the wall temperature at the beginning of this certain pipe section. In terms of Fig. 31,  $\Delta T_{max}$  is respective the temperature change from the <sup>3</sup>He pot to the first temperature clamp or from one temperature clamp to the next. From the last temperature clamp to the end of the outlet, it is assumed that the wall temperature is always at 300 K. The ratio  $\frac{\Delta T_{max}}{L}$  at each part of the outlet is listed in Table. 18.

$\frac{r}{d}$	n	$\frac{r}{d}$	n
1	20	8	24
1.5	14	10	30
2	12	12	34
3	12	14	38
4	14	16	42
6	17	20	50

Table 19: [13] The ratio between radius and diameter and its respective resistance coefficient.

## 4.3 Pressure drop in 90°-bendings

Now, we focus on the pressure drop in the two 90°-bendings, as discussed in the CDR [12]. These two bendings can, for calculation reasons, be replace by a straight cylindrical pipe with a length  $L_{eff}$  and a diameter  $D_{eff}$ . In other words, in order to receive the pressure drop we assume that the bending is a straight pipe.

## Effective Length

Firstly, the effective length of the thought straight pipe is

$$L_{\rm eff} = nD \tag{47}$$

where D is the Diameter of the duct and n is the resistance coefficient. This coefficient is dependent of the ratio between the radius of the bend and the diameter of the pipe. The obtained ratio is assigned to a certain resistance coefficient, listed in Table 19.

## **Effective Diameter**

Furthermore, for the assumption of a straight pipe instead of a bending, the diameter of the duct will transform to a effective diameter. According to [10] this transformation is only necessary for the calculation of the heat transfer coefficient h and the heat transfer Q. The effective diameter is characterized as

$$D_{\rm eff} = \frac{\pi}{2} \frac{r}{n} \tag{48}$$

where r is the radius of the bend and n the resistance coefficient.

In conclusion, the design of the <sup>3</sup>He outlet includes three straight tube parts and two bended tube parts. For the calculation of the pressure drop, the bended parts are replaced by straight tubes with a length  $L_{\text{eff}}$  and a diameter  $D_{\text{eff}}$ . Table 20: Pressure drop over an rising <sup>3</sup>He flow in the <sup>3</sup>He outlet with a diameter of 0.1 m.

$\dot{m}[\frac{g}{s}]$	0.5	0.75	1.0	1.25	1.5
$\Delta p$ [Pa]	16.5	17.8	19.5	23.1	29.8

## 4.4 Basic dependencies of pressure drop

Now we can built the final equations for the pressure drop. Because of the different flow regimes we must obtain individual formulas for laminar and turbulent flow. For the laminar regime, by Eq. (37), (39) and (40), Eq. (36) can be written as

$$\Delta p_{\rm lam} = 32 \frac{\Delta x}{D^2 A} \frac{\mu \dot{m}}{\rho}.$$
(49)

Furthermore, Eq. (38), (39) and (40) in Eq. (36) lead to a turbulent pressure drop

$$\Delta p_{\rm tu} = 0.158 \frac{\Delta x}{D^{1.25} A^{1.75}} \frac{\mu^{0.25} \dot{m}^{1.75}}{\rho}.$$
(50)

With this formulas we can obtain the pressure drop through the duct, and we see that next to the properties of the Helium, the pressure drop depends on the geometry of the duct and the <sup>3</sup>He flow inside of the duct. So following we will discuss the influence of these sources.

## Mass flow

As mentioned in the section according the cryostat design, the <sup>3</sup>He flow will change for a different amount of heat load to HEX1. An increasing amount of vapor will effect more liquid to get vaporized and respectively an decreasing amount produces less vapor. Therefore, let us look at the consequences of an changing vapor flow in the <sup>3</sup>He outlet.

Firstly, according to Eq. (40), the velocity of the helium flow will increase for an rising mass flow. In respect to Eq. (39), an increasing velocity effects a higher Reynolds number or in other words the flow regime changes as well with an increasing mass flow. For the outflow, this means that the flow is turbulent for a longer time for a higher vapor flow.

Subsequently, Eq. (42) points that an increasing amount of vapor will reduce the temperature increase of the helium in the duct. In this sense, for a larger <sup>3</sup>He flow, the vapor will heat up more slowly.

Finally, let us consider the total pressure change inside the duct over an increasing Helium flow. For an outlet with a diameter of 0.1 m the results are exhibited in Table 20. The table demonstrates that for an increasing vapor flow the pressure drop will increase as well. That means, a greater heat supply to HEX1 causes a higher vapor flow and thereby a larger pressure drop in the outlet. Consequently, the density of the vapor gets smaller at the pump and therefore, the pump needs to push more vapor through the pipe.

In summary, as the amount of flowing helium vapor increases, the overall pressure drop in the channel increases. Next to it, the flow stays in the turbulent regime for longer and the temperature of the steam will increase more slowly.

### Duct geometry

Apart from the properties of helium and its flow, the size of the outlet must be taken into account as well. The obtained Eq. (49) and (50) make it understandable that the pressure drop is heavily dependent on diameter and cross-section of the flow. In Eq. (41) we recognize that the cross-section is also a function of the diameter, because the radius is equal to half the length of the diameter. So in terms of the duct geometry, we can say that the pressure drop depends on the size of the channel diameter.

To get started, let's look at the general changes a modified geometry would bring. Regarding the flow velocity, when we use Eq. (41) in Eq. (40) and replace the radius by half the diameter, the flow velocity can be described as

$$v = \frac{4\dot{m}}{\rho\pi D^2}.$$
(51)

The squared diameter in the denominator indicates that the velocity will decrease under an increasing diameter. In addition, an decreasing velocity, effects the Reynolds number to decrease as well, because by Eq. (51) the Reynolds number, Eq. (39) can be written as

$$Re = \frac{4\dot{m}}{\pi D\mu}.$$
(52)

For the same reason as before, the Reynolds number decreases below an increasing diameter. So when the diameter gets bigger, the Reynolds number is more likely to be below 2000 or in easy words, for larger diameters, the flow regime is rather laminar.

Furthermore, the temperature change also depends on the diameter. Because the heat transfer coefficient, Eq. (44), is described by the Nusselt number, Eq. (45), which is described for turbulent flow by the Reynolds and the Prandtl number. As we have seen before, the Reynolds number becomes smaller with larger diameters and accordingly the Nusselt number will become smaller as well. However, this correlation applies only to turbulent flow, with laminar flow the Nusselt number equals 4 and is no longer dependent on the diameter.

Finally, let us consider the total pressure change over an increasing diameter. In order to see the whole influence of the diameter, the cross-section A in the obtained Eq. (49) and (50) is replaced by Eq. (41) where the radius is replaced again by half the diameter.

$$\Delta p_{\rm lam} = 128 \frac{\Delta x}{\pi D^4} \frac{\mu \dot{m}}{\rho}.$$
(53)

$$\Delta p_{\rm tu} = 1.79 \frac{\Delta x}{\pi^{1.75} D^{4.75}} \frac{\mu^{0.25} \dot{m}^{1.75}}{\rho}.$$
(54)

The diameter in the denominator has a very large influence in this equations, therefore an increasing diameter must effect an much smaller pressure drop. This can also be recognized by the obtained data in Table 21. In this chart, the results for the pressure drop are received for a constant helium flow of  $0.5 \frac{\text{g}}{\text{s}}$  and an increasing channel diameter. It can be seen that the pressure drop from a diameter of 0.06 m to 0.1 m decreases from roughly 155 Pa to almost one tenth of that value. This big difference make clear, that the diameter has a great influence on the pressure drop.

Overall, the diameter of the pipe has a big effect on the obtained pressure drop. Moreover, a bigger



Table 21: Pressure drop in the <sup>3</sup>He outlet with a constant helium flow of 0.5  $\frac{s}{s}$  and an increasing diameter of the outlet channel.

Figure 33: Part a) shows the linear increase of the vapor density under an increasing vapor pressure and part b) examines the volume flow at the pump via an increasing vapor pressure.

diameter decreases the flow velocity of the stream and therefore it is more likely to be in the laminar flow regime.

## 4.5 Effects of the pressure drop

We have seen, that the pressure drop increases with rising mass flow and decreasing duct diameter. Now, we want to look at the effects of the pressure drop on the refrigeration system.

Before discussing the effects, we first look back at the section of the cryostat design. There we talked about the vapor volume that the pump needs to push through the channel and as mentioned, a decreasing vapor density causes a higher volume. Consequently, the pump has to push more vapor or in other words the performance of the pump has to increase. A large pumping capacity requires the use of a large pump. Therefore, the density of the vapor must be kept as high as possible in order to keep the performance of the pump small.

In this sense, let us look at the behavior of the vapor density over an increasing pressure drop. An rising pressure drop means that the pressure at the pump gets smaller. This case is depicted in Fig. 33 a) and as can be seen, the density is smaller for a big pressure drop or respectively for a small vapor pressure at the pump. Consequently, the volume flow at the pump increases as shown in Fig. 33 b). In the second part of the figure it can also be seen that the closer the pressure comes to zero, the greater the volume flow increase. Consequently, a large pressure drop requires a large pump. Therefore, to keep the pump size as small as possible, a large pressure drop must be avoided.

In summary, a large pressure drop causes a large pumping volume due to the decreasing vapor density. Since the pumping speed must kept as small as possible, the aim is to avoid a large pressure drop. Table 22: Pressure drop for a pressure of 377.8 Pa at the beginning is examined in the five different sections of the <sup>3</sup>He outlet (depicted in Fig. 31), for two different helium flow rates and an increasing diameter of the duct.

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$\dot{m}[\frac{g}{s}]$	D [cm]	$\Delta p_1$ [Pa]	$\Delta p_2$ [Pa]	$\Delta p_3$ [Pa]	$\Delta p_4$ [Pa]	$\Delta p_5$ [Pa]
	6	3.9	25.2	18.9	69.1	37.5
0.5	8	0.9	7.5	5.4	18.0	8.4
	10	0.3	3.1	2.2	7.6	3.2
	6	14.1	59.5	34.4	158.6	97.5
1.0	8	2.8	10.7	6.7	23.0	11.6
	10	0.8	3.5	2.3	8.8	4.1

## 4.6 Achieving a low pressure drop

Now that we have seen the basic dependencies of the pressure drop, we want to apply the knowledge acquired to achieve an outlet design with a low pressure drop. The only condition is that the diameter of the channel at the helium bath should be as small as possible to reduce the size of the entire construction. Therefore, we want to focus on the two mentioned basic dependencies of pressure drop under consideration of the basic design of Fig. 31.

The first dependency was the mass flow within the channel and the second was the diameter of the duct, so now reference is made to an outlet related to different amounts of <sup>3</sup>He flow and different diameters of the outlet. Furthermore, as obtained in the cryostat design section, the vapor pressure at the liquid surface is about 377.8 Pa. The obtained pressure drop is deducted from this value to get the remaining pressure at the pump.

Table 22 shows the pressure drop in the different sections of the outlet for different <sup>3</sup>He flows and for different pipe diameters, so let us examine this data. Firstly, as observed before, for the same duct diameter, the pressure drop of every section is higher in the system with the bigger helium flow. Moreover, for the same mass flow, the pressure drop in every part gets smaller under an increasing diameter. Another point is that the pressure drop in the bended sections, 2 and 4, is for every adjustment higher than in the adjoining straight parts. Furthermore, the pressure drop in the first straight channel is smaller than in the following parts and the pressure drop of the first bending is smaller than the one in second bending. So we see that the pressure drop increases through the outlet.

This is obvious in Fig. 33 a), because the decreasing pressure through the outlet causes a smaller vapor density and according to Eq. (49) and (50) a smaller density means a greater pressure drop. The total pressure drop for each assumption is shown in Table 23. Here is the total pressure drop of the channel with a diameter of 0.06 m the greatest of all. At this diameter the pressure drop doubles from 0.5  $\frac{g}{s}$  to 1.0  $\frac{g}{s}$  while for the bigger diameter the pressure drop increases by a smaller friction. Thus, a larger mass flow can be handled by a larger duct diameter. Furthermore, the associated pressure and vapor density for a temperature of 300 K at the pump are listed as well and with  $\dot{V} = \frac{\dot{m}}{\rho}$  the volume flow at the pump is obtained. The results show that for a mass flow of 0.5  $\frac{g}{s}$  the volume flow increases from D = 0.06 m by about 50 %. On the other hand, for a helium flow of 1.0  $\frac{g}{s}$  the volume flow increases by roughly 2200 % from a D = 0.08 m to D = 0.06 m. This clearly shows that for a greater mass flow and a small duct diameter the performance of the pump gets to large. Because of this, let us consider how a great mass flow and a small duct diameter at the outlet of the cryostat can be handled, without effecting a too large volume flow at the pump.

Table 23: The total pressure drop in the <sup>3</sup>He outlet for two different helium flow rates and an increasing diameter of the duct. The resulting vapor pressure at the pump refers to 377.8 Pa at the beginning of the outlet and is shown with the associated vapor density and the required pumping volume.

$\dot{m}[\frac{g}{s}]$	D [cm]	$\Delta p_{\text{total}}$ [Pa]	$p_{\rm pump}$ [Pa]	$\rho * 10^{-3} \left[ \frac{\text{kg}}{\text{m}^3} \right]$	$V*10^{3}[\frac{m^{3}}{h}]$
	6	154.6	223.2	0.27	6.7
0.5	8	40.2	337.6	0.41	4.4
	10	16.4	361.4	0.44	4.1
	6	364.1	13.7	0.02	217.2
1.0	8	54.8	323	0.39	9.2
	10	19.5	358.3	0.43	8.3

Table 24: The pressure drop in a outlet that is designed with two parts of different diameters, as seen in Fig. 34. The reference pressure at the beginning is 377.8 Pa, with this value is the pressure drop for different duct diameters with its effected vapor density and volume flows at the pump stated.

$\dot{m}[\frac{g}{s}]$	$D1 \ [cm]$	$D2 \ [cm]$	$\Delta p_{\text{total}}$ [Pa]	$p_{\rm pump}$ [Pa]	$\rho * 10^{-3} [\frac{\text{kg}}{\text{m}^3}]$	$\dot{V}*10^{3}[\frac{m^{3}}{h}]$
	6		35.6	342.2	0.41	8.7
	8	10	22.2	355.6	0.43	8.4
1.0	10		19.5	358.3	0.43	8.3
1.0	6		15.5	362.3	0.44	8.2
	8	20	4.0	373.8	0.45	8.0
	10		2.0	375.8	0.46	7.9

## 4.7 Separation of the outlet

We noticed earlier, that the pressure drop increases through the outlet, in other words, the pressure drop at the end of the outlet is higher than at the beginning. Besides, Eq. (53) and (54) let us understand that the pressure drop is strongly dependent of the diameter of the duct. So since the pressure drop is the biggest at the end of the pipe, we can reduce its size by increasing the duct diameter at the end of the outlet. Consequently, we want to discuss the pressure drop for a outlet that is separated in a part with a bigger and a part with a smaller diameter.

In general, we will decrease the diameter (D1) of the first outlet segment and increase the diameter (D2) of sections 2,3,4 and 5, compare to Fig 34, with the goal of small pressure drop thorough the pipe. In this sense, we will decrease the diameter (D1) of the first part, as in the section before, from 10 cm to 6 cm and compare the results for an increasing diameter (D2) from 10 cm to 20 cm. Because of the results of the previous section, that a higher mass flows is more problematic, we will relate the pressure drop to  $0.1 \frac{g}{s}$ . The results of Table 24 show that, as expected the larger the diameters becomes, the smaller gets the pressure drop. In comparison to Table 23, the pressure drop for the worst case can be reduced by 90 % when the diameter of the pipe at the end is increased. Consequently is the vapor pressure at the pump higher and according Fig. 33 b), we can say that the volume flow at the pump gets much smaller, what requires a much smaller performance of the pump. When we compare the individual volume flows for the specific D2 assumptions in Table 24, we notice that even for the small diameter, where the pressure drop is the largest, the volume flow is almost equal. That's obvious because the pressure drop variates only in small ranges, for D2 = 10 cm between 36 Pa and 19 Pa and for D2 = 20 cm between 16 Pa and 2 Pa. Therefore, the obtained pressure at the pump is for all respective cases roughly in the same order, what results in an almost equal volume flow, compared to



Figure 34: The pipe design for a outlet that is separated in two sections with two different diameters. D1 describes the diameter of the first section of the outlet and D2 describes the diameter of the remaining pipe segments.

Table 25: The pressure drop for different outlet designs under different helium flow rates is listed. Which diameter influences which outflow section is shown in the Fig. 34. The reference pressure at the beginning is 377.8 Pa.

$\dot{m}[\frac{g}{s}]$	$D1 \ [cm]$	$D2 \ [cm]$	$\Delta p_{\text{total}}$ [Pa]	$p_{\rm pump}$ [Pa]	$\rho * 10^{-3} [\frac{\text{kg}}{\text{m}^3}]$	$\dot{V}*10^{3}[\frac{m^{3}}{h}]$
0.5	6	6	154.6	223.2	0.27	6.7(3.9)
		6	364.1	13.7	0.02	217.2
1.0	6	10	35.6	342.2	0.41	8.7
		20	15.5	362.3	0.44	8.2

Fig. 33 b).

We conclude, for an increasing diameter (D2) of the last pipe sections, the pressure drop remains, even for the smallest assumed diameter, in a small area. Consequently, the required pumping volume of the pump stays also in a small range. So for a large mass flow and a small outlet diameter at the <sup>3</sup>He bath, this represents a possibility how the outlet can be designed.

Overall, the pressure drop in an even diameter outlet becomes too large for small outlet diameters, which requires too much of the pumps. On the other hand, if the outlet is designed with a bigger diameter at the end, the initial diameter can be made smaller without causing a large pressure drop and associated high pumping power. Moreover, Table 25 indicates, that a small helium flow rate (0.5  $\frac{g}{s}$ ) is no problem for the pumps. However, at a larger mass flow (1.0  $\frac{g}{s}$ ), the pressure in the outlet drops to almost zero, what effects a strongly decreasing vapor density so that the pump has to pump a too large amount of vapor. By increasing the outlet diameter (D2) at the end, a large pressure drop can be prevented and a appropriate pumping capacity can be obtained.

# 5 Kapitza conductance

As previously mentioned, the overall aim is to produce as many UCN as possible and because of that it is essential to cool the neutrons down to a very low temperature. Thus, the helium cryostat must be able to transfers all the heat from the neutrons out. Consequently, it must be considered every step of heat conduction through the cryostat due to its expected performance. In terms of Pobell's book [14] must be the conductance at HEX 1, highlighted in Fig. 35, greatly important because the thermal boundary conductance between a solid-fluid interface gets smaller for temperatures that get closer to the absolute zero point. This conductance is called Kapitza conductance or its inverse the Kapitza resistance.

The temperature of liquid <sup>3</sup>He in HEX 1 is about 0.8 K. So the thermal boundary conductance between the superfluid and the contacted copper surface will be very low what will harm the energy output. An exemplary behavior of the affect of the Kapitza conductance is shown in Fig. 36. More precisely, the thermal boundary conductance at this crossing can dominate the heat transfer process and must be minded in order to get the thermal energy out of the UCN Guide. So for a fluid-solid interface the heat transfer q is described in reference [6] by

$$q = h_{\rm k}(T_{\rm s} - T_{\rm l}). \tag{55}$$

where  $h_k$  describes the Kapitza conductance, q is the heat flux and the temperatures of the solid and liquid are implemented by Ts and  $T_1$ .



Figure 35: The illustration depicts the schematic helium cryostat. The yellow rectangle indicates HEX1 where the issue of Kapitza conductance between liquid helium and the copper pipe at two interfaces occurs. [12]

The Kapitza conductance at HEX1 can be practical obtained by measuring the temperature of the copper and the liquid helium, the schematically construction and the temperature behavior can be seen in Fig. 37. The measured temperature difference between one of these two in combination with the known heat flux q allow to calculate the Kapitza conductance at the certain interface. In other words, Eq. (55) gets resolved after  $h_k$ .

$$h_{\rm k} = \frac{q}{T_{\rm s} - T_{\rm l}} \tag{56}$$

On the other hand, in terms of van Sciver's textbook [6] is the behavior of the Kapitza conductance described by two theories, the phonon radiation limit and the Khalatnikov theory. These two theories can be understood as the upper and lower limit for the theoretical thermal boundary conductance.

The phonon radiation limit estimates the Kapitza conductance in respect of the maximum of the



Figure 36: This Graph shows a example for the behavior of the temperature of He-II (blue line) and copper (red line) at an Cu-He-II interface. The difference between the temperatures over the time is caused due to the Kapitza conductance. [17]

phonon radiation. In other words, the Phonon radiation limit includes every kind of energy transfer by Phonons. In other words, every single Phonon in the IP helium that hits the copper surface traverses it without any reflection. Therefore, this theory is considered as the maximum of the thermal boundary conductance. The function of the Kapitza conductance described by

$$h_{\rm k} = 4\sigma T^3 \tag{57}$$

where

$$\sigma = \frac{\pi^4}{10\hbar} \left(\frac{k_{\rm B}}{\Theta_{\rm D}}\right)^2 \left(\frac{3N}{4\pi V}\right)^{\frac{2}{3}}.$$
(58)

As a result,  $h_{\rm k}$  gets

$$h_{\rm k} = \frac{4\pi^5 k_{\rm B}^2}{5h\Theta_{\rm D}^2} \left(\frac{3}{4\pi} \frac{N}{V}\right)^{\frac{2}{3}} T^3.$$
(59)

In this equations is  $k_{\rm B}$  the Boltzmann constant [26],  $\Theta_{\rm D}$  is the Debye temperature of copper [6], h is the Plank constant [26] and  $\frac{N}{V}$  describes the molar volume of copper [27]. Because of no liquid properties in this equation, the Kapitza conductance is for <sup>3</sup>He the same as for <sup>4</sup>He. All constant values of Eq. (59) together lead to

$$h_{\rm k} = 4484 \frac{\rm W}{\rm m^2 K^4} T^3.$$
(60)

In summary, the value for the Kapitza conductance isn't dependent on the liquid at the interface but it is very sensitive to the temperature due to its proportionality to the temperature to the power of three.



Figure 37: The figure depicts above the schematic cross section of HEX1 with one IP-He-Cu and one Cu-3He interface. The heat flux is depicted from IP helium towards the <sup>3</sup>He. The lower diagram shows the behavior of the temperature through the upper cut including the temperature changes caused by the Kapitza resistance. [17]

Therefore, the value for the thermal boundary conduction gets at very low temperatures significant small. However, because of the reason that in this case ideally all phonons considered to cross the copper transition it can be assumed that the phonon radiation limit is overestimating the Kapitza conductance.

On the other hand, the Khalatnikov theory is the lower limit for the thermal boundary conduction. This way described by Swartz and Pohl [15], uses the low temperature analogy of phonons with photons, the particles of classical boundary scattering in optics. So the Khalatnikov theory considers the reflection of phonons at the surface between the two media like the reflection of photons in classical optics. Van Sciver explains in his textbook [6] that in this theory phonos only cross the section if its angle of incidence is smaller than a little critical angle. phonons that impinge in a larger angle will be reflected. This way of characterizing the Kapitza conductance is similar to the photon radiation limit expect that the reflection of phonons is taken in account. Therefore Eq. (57) from the phonon radiation limit can be taken over. The reflection of phonons is obvious in factor  $\sigma$ , which can be described for a small temperature difference at the interface as

$$\sigma = \frac{3\pi^4}{10} \frac{RF\rho_{\rm L}c_{\rm L}}{M\Theta_{\rm D}^3} \tag{61}$$

So the Kapitza conductance is expressed as

$$h_{\rm k} = \frac{6\pi^4}{5} \frac{RF\rho_{\rm L}c_{\rm L}}{M\Theta_{\rm D}^3} T^3,\tag{62}$$

where  $\rho_{\rm L}$  is the density and  $c_{\rm L}$  is the sound velocity of the Helium which are obtained by HePak and He3Pak [22, 23]. The quantity R is the ideal gas constant with a value of 8.31  $\frac{\rm J}{\rm molK}$  [26] and  $M = 64 \frac{\rm g}{\rm mol}$  characterizes the molecular weight of copper. The factor F describes the transmission and reflection of the phonon at the solid and liquid interface. Reference [6] finds F equals 1.6. As shown in Fig. 37, at HEX1 is one transition between <sup>4</sup>He and copper and the other one between <sup>3</sup>He and copper. Due to differences in density and sound velocity of the two types of helium, different equations are obtained for the use case on HEX1.

All properties of liquid  ${}^{4}$ He and copper used in Eq. (62) lead to

$$h_{k4} = 21 \frac{W}{m^2 K^4} * T^3.$$
(63)

The same procedures for <sup>3</sup>He and Copper gives

$$h_{k3} = 8\frac{W}{m^2 K^4} * T^3.$$
(64)

So the Kapitza conductance at the IP helium side is almost 3 times bigger than on the <sup>3</sup>He side. That is because the density and sound velocity are both bigger for <sup>4</sup>He.

With the assumption of the same temperature at both interfaces, the ratio between the thermal boundary conductances at the different helium interfaces is

$$\frac{h_{\rm k4}}{h_{\rm k3}} = 2.6. \tag{65}$$

So under this condition, the conductance at the  ${}^{4}$ He-Cu interface is 2.6 times bigger than that at the conductance at the  ${}^{3}$ He-Cu interface.

In summation, there is a clear difference in conductivity between these fluids in this theory. Due to its similarity to the phonon radiation theory is the Kapitza conductance is in this case also strongly dependent on the temperature. Nevertheless, because of the transmission of only a small angle of incidence phonons, the Khalatnikov theory is considered an underestimate of the Kapitza conductance.

All in all, the Kapitza conductance described by the Phonon radiation limit and the Khalatnikov theory is in both cases strongly depended on the temperature. So at very low temperatures the value for the Kapitza conductance gets rapidly smaller. Furthermore, the factor of the Khalatnikov theory is about 200 times smaller due to the minded reflected phonons at the interface between the liquid and solid. This reflection means that thermal energy can't leave the medium and stays in the Helium. According to the aim, to bring the thermal energy out of the UCN guide, the scattering makes it difficult to reach this goal. Hence, the next task is to estimate how this factor of the Kapitza conductance at HEX1 is affected.

## 5.1 Surface morphology

In the last pages the theoretical limits for the Kapitza conductance were shown. Thereafter, in order to get even closer to the right value for the thermal boundary conduction at HEX 1, it is needed to get a closer look at this region between the Phonon radiation limit and the Khalatnikov theory.

Further experiments concluded that the texture of the surface between the two media plays a major role in the heat conduction [16]. Surfaces that are cleaned well either chemically or mechanically have a good heat conductance, compare to the green line displayed in Fig. 38. In contrast, surfaces that are bad prepared and are less cleaned, the Kapitza conductance is worse, like the purple line in Fig. 38 indicates. Consequently, with the aim to estimate the Kapitza conductance at HEX1, an factor that describes the goodness of the surface must be implemented to the equations of the Kapitza conductance. Therefore a factor  $\kappa_{\rm G}$  is added to the equations of the Khalatnikov theory.

$$h_{k4} = \kappa_{\rm G} * 21 \frac{\rm W}{\rm m^2 K^4} * T^3 \tag{66}$$

$$h_{k3} = \kappa_{\rm G} * 8 \frac{\rm W}{\rm m^2 K^4} * T^3 \tag{67}$$

In order to know, how big  $\kappa_{\rm G}$  approximately will be, the value for the upper and lower limit from the experimental data, depicted in Fig. 38 will be used to get an estimation of the factor  $\kappa_{\rm G}$  according a Cu-4He interface. Associated solutions for the factor of the surface are  $\kappa_{\rm G} = 8.5$  for the lowest measured Kapitza conductance and  $\kappa_{\rm G} = 53.6$  is the factor for for best attempt. Therefore, a factor of the surface goodness  $\kappa_{\rm G}$  for the cryostat can be expected of roughly 8.5 to 53.6. In this context, a higher  $\kappa_{\rm G}$  is equal to a better prepared surface and consequently a better thermal boundary conductance. In consideration of this  $\kappa_{\rm G}$ -range, the Kapitza conductance can be assumed for bad <sup>4</sup>He-surfaces to be in the order of

$$h_{\rm k} = 179 \frac{\rm W}{\rm m^2 K^4} * T^3 \tag{68}$$

and for well cleaned surfaces to be

$$h_{\rm k} = 1126 \frac{\rm W}{\rm m^2 K^4} * T^3.$$
(69)

Besides, with the aim of estimating the Kapitza conductance at HEX1, the partner organization from Japan [17] simulated this transition under real conditions. The data resulted in an expectation of  $\kappa_{\rm G}$  equal to 43 what indicates a good texture of the surface. Because of that, the equations for the Kapitza conductance can be written as

$$h_{k4} = 903 \frac{W}{m^2 K^4} * T^3$$
(70)

and

$$h_{k3} = 344 \frac{W}{m^2 K^4} * T^3.$$
(71)

It is important to mention that this equations are only assumption. The texture of the surface can vary across the entire area. Therefore the Kapitza conductance can vary as well. As a result of this, the Kapitza conductance should be more thought of a range than a certain value.



Figure 38: Measured Kapitza conductance at temperatures from 1.3K to 2.0K at an Copper-He-II interface with different surface conditions. The red filed area shows the region where the measurements of Kapitza conductance at different surface conditions were. The green line shows measurements of Kapitza conductance for a very well cleaned surface and the purple line measurements of Kapitza conductance for a very bad surface condition. The yellow line refers to experimental data, that simulated the HEX in order to get a estimation for Kapitza conductance that is possible at TRIUMF [17]. The blue line shows the theoretical upper limit of the Kapitza conductance, called phonon radiation limit, the red line shows the theoretical lower limit, the Khalatnikov theory. [16, 17]



Figure 39: The temperature measurements in a  ${}^{3}$ He bath and a  ${}^{4}$ He bath over a variating heat input are displayed. The non-filled circles are equal to the measurement points M3 and the filled ones regard to the measurement point M4. [19]

#### 5.2 Comparison of previous results

Measurements at the old cryostat design at TRIUMF can be analyzed referring the temperature change at HEX1. These results must be benchmarked against another similar heat exchanger to test their validity. In the following section, comparisons with a heat exchanger of a dynamically polarized proton filter experiments are shown.

## 5.3 Heat exchanger of a dynamically polarized proton filter

This heat exchanger is described in Ref. [19]. In this case, the heat flux was from <sup>3</sup>He through a copper layer to He-II implemented, compare to Fig. 37, and then the temperatures at both Helium states were measured as a function of heat input, shown in Fig. 39. With the aim of maximizing cooling performance, the surfaces were increased by creating the HEX with small fins on both sides similar to the design in Fig. 40. So the surface areas got about 300 cm<sup>2</sup> large. Upon a closer look at the values at a heat input of 35 mW the temperature at of the <sup>3</sup>He was 460 mK and the temperature of the He-II was 500 mK. Consequently the temperature difference was 40 mK. The theoretical change in temperature is obtained through the transformation of Eq. (55) to

$$\Delta T = \frac{q}{h_{\rm k}}.\tag{72}$$

The heat flux q is the total power crossing the interface divided by the surface area

$$q = \frac{Q}{A} \tag{73}$$



Figure 40: The design of the old Copper heat exchanger with fins on both sides, it was used in the vertical UCN source. [20]



Figure 41: The old design of HEX1 is displayed schematically. The upper point M3 is the temperature measurement point in the <sup>3</sup>He bath and the lower point M4 is the measurement point in UCN guide. The orange part between the two helium section points the copper pipe and the red arrow indicates the heat flux. [18]

and the Kapitza conductances can be estimated using Eq. (70) and (71). The calculation of the Cu-He-II interfacial temperature difference resulted in  $\Delta T_{3,Cu}=35$  mK and  $\Delta T_{4,Cu}=11$  mK. So it gives a total prediction of  $\Delta_{4,3}T=46$  mK.

In summation, the size of the calculated temperature change is reasonable to think, because it is in the right order of magnitude and has only a small derivation which can be caused by a different surface texture than expected.

## Heat exchanger of the vertical UCN source

This design of the old heat exchanger, shown in Fig. 40, is described by Masuda [20]. In this construction, the surfaces on both sides were increased to  $2600 \text{ cm}^2$  by fins. The obtained data, depicted in Fig. 42, is obtained at the measurement points as in Fig. 41 demonstrated. In these measurements, a distinction is made between "base" measurements and "sat" measurements. The "base" values are prior the activation of the heater and the "sat" measurements are after saturation of the temperatures with the heat on. The "sat" state is usually reached after several minutes of heat application when the <sup>3</sup>He flow became stable. Reference [18], the temperature difference at saturation was 423 mK under a supplied



Figure 42: The chart shows the result of temperature measurements from last November at TRIUMF. The praph depicts the behavior of the temperature difference between two measurement points, one inside of the UCN guide and one in the <sup>3</sup>He section, over different an increasing amount of heat input. The "base" values are regarding to the activation of the heater. The "sat" values mean after saturation of the temperatures with the heater on. [18]

heat of 250 mW.

Calculating the Cu-He-II interfacial temperature difference at a <sup>3</sup>He temperature of 0.94 K like in the heat exchanger before yields to  $\Delta T_{3,Cu}$ = 3.4 mK and  $\Delta T_{4,Cu}$ = 1.3 mK. In summation, a theoretical temperature change of 4.7 mK at HEX1 is obtained.

So the theoretical result is about 100 times smaller than the measured value from November.

A cause of that could be the temperature raise due to the thermal resistance of the pipe. Therefore the temperature change through the Copper pipe is taken in account. Regarding Ref. [24], the temperature difference across the Copper is

$$\Delta T = \frac{Q\Delta x}{Ak_{\rm Cu}} \tag{74}$$

where Q is the applied heat,  $\Delta x$  is the thickness of the heat exchanger without fins, A is the surface area of the Copper and  $k_{\rm Cu}$  is the thermal conductivity of Copper which equals 1.8  $\frac{\rm W}{\rm cmK}$ . In this case, the small fins of the heat exchanger are neglected and a surface at the <sup>3</sup>He of about 78.5 cm<sup>2</sup> is assumed. This calculation results in a temperature change of 2 mK. So the Kapitza conductance and the Copper conductance yield to a total change of 6.7 mK. This result does not approximate the measured value, so there must be another cause. It might happen that the <sup>3</sup>He refill was to little, so that cooling bath run out of liquid for its cooling purpose. Consequently, the required cooling can't be delivered

## 5.4 Heat transfer regimes

Another even more important cause could be a change in the boiling regime. As mentioned in Ref. [25], when the heat transfer regime alternates, the temperature change between the liquid and the solid is significantly affected.



Figure 43: Part a) depicts schematically the heat transfer with natural convection at a surface where heat is applied from underneath to the surface. In this case, warmer liquid He-I is flowing up and mixing with the colder section above. Next to it, section b) shows the nucleate boiling regime at the same experimental procedure with a larger amount of heat supply. Here are forming bubbles at the heated surface which start rising to the top. Item c) is film boiling which occur under even higher energy input. Because of the large energy input, bubbles are forming and detaching very fast. So that they form the pictured vapor film between surface and He-I. [6].
Firstly, van Scivers [6] studies show, the heat transfer by natural heat convection, shown in section a) of Fig. 44, is when the liquid gets heated without any boiling, at the surface the density of the liquid increases by heating and therefore the liquid moves up. The up moving helium creates new space for colder helium with higher density to replace it.

If the heat input is increased the heat transfer regime changes to nucleate boiling, part b) of Fig. 44. In this circumstances forms a superheated liquid layer right on the Copper surface and because of that small bubbles appear at the surface of the copper which start rising to the top.

Lastly, for an even higher energy input the system switches to film boiling, the third segment of Fig. 44. Because of the large energy input, the bubbles are getting bigger and form and detach very fast. As a result, the bubbles fuse together and form a continuous vapor layer between the solid and liquid helium.

Due to the high energy input at the <sup>3</sup>He interface of HEX1, it is likely that nucleate boiling or film boiling occurs and the following analysis show that the temperature change varies between the individual systems.

Reference [25] presented experiments regarding this two regime transitions.

In this analysis was an increasing amount of heat brought to a Copper surface while measuring the temperature difference between the Copper surface and an adjacent bath of liquid <sup>3</sup>He, which is to begin at 0.8 K. The observations with an decreasing amount of heat input is unattended in this section. In both parts of Fig. 44 are the results of these experiments demonstrated. The pictures illustrate the temperature difference in each of the three different regimes. With low added heat, the temperature difference at natural heat convection is rising in an almost linear shape. After putting in more heat, a large discontinuation of the temperature change appears. In other words, despite increasing the heat supply, the temperature change decreases suddenly. At this point changed the regime from heat convection to nucleate boiling. After this transition is the function of temperature again rising almost linearly, just with a different slope. Moreover, with further increase of the heat flux, another discontinuation arises. The temperature change varies greatly again and gets abrupt bigger. This marks the second regime change. In further consideration of part a) in the Fig. 44, it is demonstrated that at the first regime change the temperature difference at the same heat input is more than halved. Furthermore, in the more likely case when changing from nucleate boiling to film boiling, the temperature is increased almost ten times. Therefore, the measured value of the temperature change at HEX1 is strongly dependent on the heat transfer regime in the  ${}^{3}$ He and could explain the high measured temperature change from the last heat tests.

Moreover, part b) of Fig. 44 is the graph with the neglected amount of temperature change caused by Kapitza conductance. This plot shows a temperature change that is a few milli-Kelvin smaller than before. This temperature change is consistent with the results of temperature change received earlier.

To conclude, the theoretical change in temperature of 6 mK and the measured temperature difference of roughly 400 mK are to dissimilar. So the measurements can't be explained by Kapitza conductance but a few things can be concluded from the November heat tests.

First of all the difference in base temperatures can't be explained by the Kapitza conductance. This difference always seems to be approximately 0.18-0.2 K in every measurement. Even if the worst case for Kapitza conductance, the Khaltnikov theory is applied the temperature difference wouldn't reach such high values. To check if the temperature offset is the issue, the values of the temperature sensors are compared to vapor pressure measurements at similar positions. Because the temperature can be received by HePak [22] and He3Pak [22] at known pressure. However, the obtained data by the pressure sensors isn't significant due to fixed baffles at the <sup>3</sup>He outlet. The baffles have the purpose to



Figure 44: [25] The diagrams show the temperature difference between Copper and <sup>3</sup>He during heat input to the Copper side. The experiment was under saturated vapor pressure. Part a) is the temperature difference including the Kapitza conductance and part b) is the temperature difference without the proportion of the temperature change caused by Kapitza conductance. The blue dots refer to the regime of natural heat convection, the red points are according the nucleate boiling regime and the green dots at the top belong to a third regime, the film boiling. The empty dots refer to the same experiment with an decreasing heat input, but this observation is neglected in this context. It is noticeable that a switch in regime a significant change in temperature causes. The arrows between different point of separate regimes show that the temperature change for the same applied heat isn't the same for different regimes.

limit radiation heat in the pipe but as a consequence, the pressure recorded to low what causes an inaccurate temperature estimation. Hence, it is difficult to explain the whole obtained data.

Secondly, the Kapitza conductance offers no explanation for the rise of the temperature difference by applied heat above 200 mW. Maybe this is caused by a lack of cooling power. In other words the pumping speed of the <sup>3</sup>He could be too small or more precisely the <sup>3</sup>He could get vaporized to fast. Consequently, too less cooling power is provided and as a result of this the <sup>3</sup>He could be heated to much. Moreover, it may occur that the the boiling regime changes from nucleate to film boiling. Under this condition, the sudden temperature change can be explained by this topic. Therefore, another further task will be to determine the heat transfer regime to be able to make a clear statement about this abrupt temperature change.

Furthermore, Kapitza conductance could explain the small differences between the base and saturated values by lower applied heat. For this case the temperature difference in the base measurements can assumed to be caused by calibration errors of the temperature sensors. However, this data needs to be measured more carefully in order to make a more accurate statement.

In summary, the first heat exchanger of the dynamically polarized proton filter seems to be in good agreement with Kapitza conductance. It is reasonable to think that the whole temperature change is caused by Kapitza conductance. In addition, the results of the Okayama University confirm the calculated temperature change by Kapitza conductance. Secondly, the heat exchanger of the vertical UCN source has some inconsistencies. Only the relative temperature changes at a low applied heat power are in agreement with the Kapitza conductance. So it must be a further task to measure the temperature more precisely to get a accurate statement about the Kapitza conductance at HEX1. At last, the heat transfer regime needs to be received during the process, in order to be able to classify the temperature change.

## 5.5 Kapitza conductance at the new HEX design

All analyzes of Kapitza conductance are regarding the former cryostat design. Due to the development of a new cryostat design, HEX1 changed as well. So this section is about the estimated temperature change at the new design of HEX1.

The way of the heat in HEX1 is schematically shown in Fig. 45 and roughly said it is the way form the IP helium through a round Copper pipe with a Nickel coating on one side and at the end to the <sup>3</sup>He bath at 0.8 K. This route can be separated in different parts of heat conduction. This sections are explained vise versa, so from the <sup>3</sup>He side to the <sup>4</sup>He side. Thus, because the temperature of the <sup>3</sup>He can be obtained by the precooling, but the temperature of the IP helium needs to be estimated because it can't reach the same temperature as the <sup>3</sup>He by cause of thermal boundary resistance.

At first there are the Kapitza conductance at the <sup>3</sup>He-Cu interface. The liquid helium is at 0.8 K and therefore the Kapitza conductance at this interface can be calculated regarding equation 71. The heat Q that is transferred is divided up on the surface A of the cylindrical pipe like presented in equation 73. The surface of a cylinder is known as

## $A = 2\pi r L$

where r is the radius and L the length of the cylinder. Regarding the new HEX design is the IP helium side of the HEX a regular cylinder without any fins. On the other hand, is the  ${}^{3}$ He side planned with



Figure 45: The figure shows the schematic cross section at HEX1 without fins. On the left hand side are the materials of the pipe, Copper and Nickel, and the mediums of the of the different parts. The heat flux is pointed by the red arrow. The green point (M4) is a temperature sensor in the UCN guide and the blue point (M3) is a temperature sensor in the <sup>3</sup>He pumping circle. [18]



Figure 46: Part a) illustration of the cross section of the cylindrical HEX. The blue circle refers to the inner radius and inner surface of the HEX. The outer green circle depicts the outer radius and surface. The fins around the duct are indicated by the green circle. The view in section b) shows the HEX from the side with the fins that are like rings around the main tube.

fins on the surface, as illustrated in Fig. 46. The surface area of the cylinder with fins equals three times the surface area without fins.

Moreover, the temperature on the inside of the Copper surface can be obtained by Eq. (55).

$$T_{\rm Cu} = \frac{q}{h_{\rm k}} + T_{\rm 3He} \tag{75}$$

Using the equations above, the temperature at the <sup>3</sup>He-Cu interface at the Copper side gets 0.88 K. The next step is to estimate the heat loss through the cylindrical Copper pipe. In terms of Ref. [11], for this estimation it is only necessary to consider the bulk of the HEX without any fins. According reference [21] the temperature at the inside is of a cylindrical pipe is stated by

$$T_{\rm Cu1} = QR_{\rm Cu} + T_{\rm Cu2} \tag{76}$$

where  $T_2$  is the outer surface temperature and  $T_1$  is the inner surface temperature of the copper, Q is the heat input and  $R_{Cu}$  is the heat resistance of the Copper. The resistance is characterized via

$$R_{\rm Cu} = \frac{r_2 - r_1}{k_{\rm Cu} A_{\rm m}} \tag{77}$$

where  $r_1$  is the inner radius and  $r_2$  is the outer radius as depicted in Fig. 46,  $k_{Cu}$  is the conductivity of the Copper and  $A_m$  is the mean area of the interface. The mean area is given by

$$A_{\rm m} = \frac{A_2 - A_1}{\ln(A_2/A_1)} \tag{78}$$

where  $A_1$  is the inner surface and  $A_2$  the outer surface without fins as shown in Fig. 46.

In summation, the temperature of the Copper, with a length of L = 0.5 m and the two radii  $r_1 = 0.15$  m and  $r_2 = 0.14$  m, rises of 0.7 mK. This small size is reasonable due to the good heat conductivity of Copper. Furthermore, the temperature rise through the tiny layer of Nickel can be assumed as 0 due to its negligible size of less than 1 mm.

At last, the transition from Nickel to IP helium must be considered again. This case is similar to the first calculation with Kapitza conductance referring equation 75 just with the consideration of Eq. (70). However, the higher temperature in this case provides a higher Kapitza conductance. In this context, the IP helium temperature at the inside of the pipe reaches a value of 0.95 K.

All in all, the estimated temperature rise caused by Kapitza conductance is 0.15 K. At the <sup>3</sup>He interface is the temperature increase with 0.08 K almost equal to the change of 0.07 K at the IP Helium interface. Thus, because both interfaces are at almost the same temperature due to the good conductivity of Copper and the negligible layer of Nickel. Moreover, it is even more important that the three times bigger surface at the <sup>3</sup>He side is compensated by the almost three times greater Kapitza conductance at the <sup>4</sup>He side.

However, the Kapitza conductance has a high uncertainty like shown in Fig. 38 and for this calculation is the Kapitza conductance roughly estimated. So for an assumption of a very good cleaned surface or a very bad surface, it is possible that the temperature change can be 0.12 K at the best case or 0.55 K at the worst case. Therefore it may be that for bad surface conditions the real temperature increases is about four times the received value. In order to get more certainty about this big uncertainty are scientist of the collaborating institute in Japan are still working towards a good fitting approximation for the Kapitza conductance.

In conclusion, the Kapitza conductance is the thermal boundary conductance between liquid helium and a solid surface. Since these conductance gets worse for lower temperatures, Kapitza conductance must be considered at HEX1 where the conductance at two liquid-solid transitions could harm the heat transfer. For this consideration we used the Khalatnikov theory and the Phonon radiation limit to limit the Kapitza conductance and obtained with the influence of the surface morphology a equation for the thermal boundary conductance. Thereafter, the obtained equation applied on the heat exchanger of a dynamically polarized proton filter showed us that the equations are good description for the Kapitza conductance. So we were able to discuss the measured temperature behavior at the old design of the heat exchanger. The measured results were much bigger than the calculated results , what could have different reasons. The most important cause seemed the heat transfer regime, because different heat transfer regime from the last measurements was not clear at all, no clear statement can be made about how big the influence of Kapitza conductance in the last measurements was. So a for further analyzes of the temperature change caused by Kapitza conductance, the heat transfer must be obtained.

## 6 Conclusion

All in all, for the design of the new UCN source, we looked at various topics related to the helium cryostat at Triumf. Let us conclude by capturing the most important results of each section in short sentences.

We started with the current design of the helium cryostat. The first subject was the performance of the several counter flow HEX. On the whole, HEX4, HEX5 and HEX6 are important steps for the pre-cooling of <sup>3</sup>He and for the current assumptions with two upstreaming cooling paths each of them can deliver more than the required amount of cooling. However, since one of these counter flowing paths applies the required cooling almost by itself, the importance of the second path is questionable. For a removal of this second path, the cooling capacity of the second path must be increased by a small amount. However, since this work contains only a rough analysis on this topic, this subject should be part of further studies. Next to it, the performance of HEX2, HEX3 and HEX4 was examined for their importance. The discussion ended with the finding that HEX2 and HEX3 are negligible due to their lesser importance. HEX2 would increase the heat load to the <sup>3</sup>He bath what results in a increase of roughly 800  $\frac{g}{s}$  of the <sup>3</sup>He flow rate. Besides, the removal of HEX3 would lead to a <sup>4</sup>He consumption of roughly 81  $\frac{L}{h}$  in the pre-cooling process of <sup>3</sup>He. Both of these removals would mean a larger helium consumption and therefore, a larger performance of the pumps. HEX4 is a very important HEX, since its performance heavily influences the <sup>4</sup>He consumption in the 4K bath. A very bad performance of this HEX means that the total <sup>4</sup>He consumption at the inlet can increase to up to 91  $\frac{L}{h}$ . So the poor performance of HEX4 must be considered for the final decision about the cryostat design, because for a bad performance of HEX4 enough <sup>4</sup>He must be available to compensate its missing cooling. The worst case is for a removal of HEX2 and HEX3 and a very poor performance of HEX4. In this case can the total <sup>4</sup>He consumption rise to up to 131  $\frac{L}{h}$ 

Thereafter, because the pressure drop greatly affects the required performance of the pump, we discussed its dependence and prevention. In general we noticed that the pressure drop for high evaporation rates and a small duct diameter can not be handled. As a result, we changed the design of the outlet and separated it in two sections of different diameter. So that the pipe right at the helium bath is designed with a small diameter and the following outlet with a greater diameter. In summation, for an outlet design like this, the great mass flow and the small duct diameter can be compensated, so that for the worst case the pressure drops only by 16 Pa instead of 364 Pa. Therefore, the need for high pump performance can be avoided.

At the very end, the occurrence of the Kapitza conductance was examined. In summary, Kapitza conductance is the conductance between liquid helium and a solid surface. This conductance will decrease with decreasing operating temperature and hinder the heat transport thereby. In order to make an assumption about this conductance, we needed noticed that the surface morphology has a great effect on the conductance. Assumption from the collaborators form Japan allowed to estimate this influence and allowed to obtain an equation for the Kapitza conductance. The application of this equation on a similar built of HEX confirmed the accuracy of this equation. Following, the results of further temperature measurement at the old HEX design and the theoretical obtained data showed that both are in a huge disagreement. The measurements were with values around 400 mK to large compared to the prediction of 5 mK by the calculations. So we understand that other sources like the heat transfer regime in the helium bath could have caused the large temperature change. However, since the heat transfer regime wasn't obtained in the last measurements, no proper statement of the

influence of these source can be made. Therefore, a further task must be to obtain the heat transfer regime in the helium bath, to be able to analyze the temperature change in more detail.

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